

Soil Water Models

A Review



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Soil water models

A review

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Abstract

The development of soil water models, including those which deal with soil water uptake by plant roots, are reviewed and compared. Major assumptions employed in the three types of models, the physical based-, the budget-, and the combination ones, are discussed. The physical based- and the combination models can be easily adopted to a wide range of soil, climate and crop conditions. Their major drawback of requiring extensive knowledge of soil and crop characteristics limits the widespread use of this approach at the present time. On the other hand, the budget models require a low level of input, but need to be recalibrated if they are used in environments other than those for which they were derived. Suggestions are made regarding future use if the models are employed for areal soil water estimates.

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I. Introduction

The importance of water in plant growth and crop development is widely accepted. Many yield estimates are based on the correlation between available soil water and yield. Furthermore, many crop growth simulation models rely extensively on detailed information of the soil water regime throughout the growing season.

For many other purposes a calculation of the effect of precipitation on the water conditions of the soil is needed. For example how will the bearing capacity of different soils be influenced for traffic or cattle by precipitation? What will be the amount of water stored on the surface or the amount of runoff under natural precipitation patterns? How will these be affected by soil improvements or drainage?

All common techniques to measure soil water have inherent shortcomings and require considerable calibration and replication to provide representative soil water data. Most direct readings of soil water provide only point readings and do not integrate the measurements over space and time as is often required for crop growth and yield estimations, runoff calculations, drainage calculations, etc. To overcome these difficulties, mathematical modeling and simulation techniques, relying on the use of high speed computers, have been developed for the purpose of providing a comprehensive quantitative description of the behaviour of the soil-water-plant system.

Different approaches have evolved in describing the fate of water entering and moving through the soil. The physical based models use the principle of continuity and Darcy's law to derive the soil water flow equation with water moving through the soil in response to a water potential gradient. On the other hand, soil water budget models estimate the daily balance of soil water from empirical functions for separate components of the system. Recently several researchers have modeled soil water using a combination approach, i.e. incorporated several aspects of the budget models into the physical based models or vice versa.

The purpose of this report is to review and compare the development of soil water models, including those which deal with soil water uptake by root systems. The models will be assessed with regard to their potential suitability of estimating regional soil water conditions and water use by agricultural crops.

II. The physical based models

A. Mathematical formulation.

The theory for transient, isothermal flow of water in a nonswelling soil can be described by a combination of two equations:

- (1) Darcy's law, which states that the flux of water (q) is proportional to, and in the direction of, the driving force which is the effective potential gradient:

$$q = -K \nabla \phi \quad (1)$$

where ϕ , the hydraulic potential, is the sum of the matric potential (ψ) and the gravitational potential. Expressed in head units (free energy per unit weight), the hydraulic potential can be written as:

$$\phi = \psi - z \quad (2)$$

where z is the gravitational level expressed as depth below the soil surface. K is the hydraulic conductivity, which in the unsaturated soil can be expressed as a function of water content, θ , or matric potential ψ .

- (2) The continuity equation, which states that the time (t) rate of change of water content in a volume element of soil must equal the divergence of the flux:

$$\partial\theta / \partial t = \nabla \cdot q \quad (3)$$

These two equations are combined to give the general soil water flow equation:

$$\partial\theta / \partial t = \nabla \cdot (K \nabla \phi) \quad (4)$$

which, if the system is considered as vertical, and the z direction is taken as positive from the soil surface downward, becomes:

$$\partial\theta / \partial t = \partial/\partial z \{K (\partial\psi/\partial z - 1)\} \quad (5)$$

To simplify the mathematical and experimental treatment of unsaturated flow processes, the soil water diffusivity has been defined as (Childs and Collis-George, 1950):

$$D(\theta) = (K(\theta)/C(\theta)) \quad (6)$$

where

$$C(\theta) = d\theta / d\psi \quad (7)$$

is the specific water capacity. Equation (5) can then be written as:

$$\partial\theta / \partial t = \partial/\partial z (D(\theta) \partial\theta/\partial z) - \partial K(\theta)/\partial z \quad (8)$$

The advantage of using the diffusivity equation lies in the fact that the range of variation of diffusivity is smaller than that of conductivity, which consequently facilitates more accurate averaging procedures. Furthermore, the wetness and its gradient are often easier to measure than the matric potential and its gradient. On the other hand, equation (8) is restricted to the class of problems in which the matric potential changes monotonically, because it fails to take into account hysteresis effects in the relations $\psi(\theta)$ and $K(\theta)$.

If both saturated and non-saturated regions in the soil profile are of interest, it is better to consider ψ instead of θ as the independent variable (Philip, 1958). Using the specific water capacity, equation (5) is then transformed into:

$$C(\psi) \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z} \{K(\psi) \left\{ \frac{\partial \psi}{\partial z} - 1 \right\}\} \quad (9)$$

where ψ varies from positive values in the saturated zone to negative values in the unsaturated zone.

In order to describe the upward movement of water from a phreatic surface (called capillary rise) the flow equation can also be formulated in terms of steady state conditions, as opposed to transient conditions which were discussed above. Under steady state conditions the water content at any particular point does not change with time and the equation of continuity reduces to:

$$dq/dz = 0 \quad (10)$$

which upon integration gives

$$\bar{q}_c = \bar{q}_c = \text{constant} \quad (11)$$

where \bar{q} is the steady vertical flux given by Darcy's law:

$$\bar{q}_c = -K(\psi) \frac{d\phi}{dz} \quad (12)$$

Substituting the sum of matric and gravitational potential for the hydraulic potential (eqn. 2), the equation for steady flow can be written as:

$$dz = K(\psi) d\psi / (K(\psi) - \bar{q}_c) \quad (13)$$

Taking the reference level at a stationary phreatic surface at which $z = z_0$ and $\psi = 0$, solving for z yields:

$$z = z_0 + \int_0^{\psi(z)} [K(\psi) / (K(\psi) - \bar{q}_c)] d\psi \quad (14)$$

which defines the height above the water table at which a given steady upward flux can be maintained for a given matric potential at this height. A systematic application of equation (14) to compute heights of capillary rise for different values of the flux \bar{q}_c was first carried out by Wind (1955).

The above mathematical formulation of the soil water flow equation is given in terms of the one-dimensional, vertical case. However, the formulation can be expanded to include the second and third dimension (see e.g. Freeze, 1971). This, in principle then would make the physical based models suitable for assessing areal soil water conditions.

B. Methods of solution

Under field conditions the description of soil water movement is highly complicated since the initial and boundary conditions are usually not constant. Also the soil properties change

throughout space and sometimes vary with time. In view of this, most efforts have in recent years been concentrated on seeking numerical, as opposed to analytical solutions of the soil water flow equation.

Digital simulation of water flow in soils inherently leads to a division of the soil profile into a number of compartments of some specified thickness. Irrespective of the kind of simulation performed, it is accomplished by calculating fluxes over finite time steps and using these to estimate changes in soil water content. Due to the nature of simulation, one has no alternative but to keep the fluxes constant during an individual time step. Thus, one assumes a situation of steady state between two points in the soil at some finite distance $z_2 - z_1 = \Delta z$ and approximates Darcy's law (eqn. 1) by:

$$q' = \overline{K(\theta)} \Delta\psi / \Delta z \quad (15)$$

where q' is now the flux across the boundary between the two compartments, and $\overline{K(\theta)}$ is an average hydraulic conductivity between z_1 and z_2 , where the water contents are θ_1 and θ_2 respectively. The potential gradient is estimated by dividing the finite difference $\Delta\psi$ by Δz .

Apart from the above assumptions one has to deal with the problem of choosing an appropriate time interval over which the calculations are to be made. Obviously, the larger the time interval, the poorer the estimate of water movement. In actual fact, if the time intervals are chosen too large, relative to the size of the compartments and their water contents, the model becomes unstable and the calculation results begin to oscillate (Van Keulen and Van Beek, 1971). On the other hand, if the time interval is very small, one soon encounters the practical problem of excessive computer time requirement. This problem can be partly minimized by choosing a versatile simulation language called Continuous System Modeling Program, CSMP (Speckhart and Green, 1976), which uses a variable time interval method of integration. Such a program chooses the largest time interval permissible for a given error. This system also has the advantage that the time interval can be changed as the simulation proceeds, thus avoiding small time intervals during stages of the simulation where they are not needed.

Various numerical methods have been developed to solve the physical based models. In the explicit method (Staple, 1966) a series of linearized independent equations is solved directly, while in the implicit method (Hanks and Bowers, 1962; Molz and Remson, 1971) a system of linearized equations has to be solved. For a given grid point at a given time, the values of the coefficients $C(\psi)$ and $K(\psi)$ can be expressed either from their values at the proceeding time interval (called explicit linearization) or from a prediction at time $(t + \frac{1}{2}\Delta t)$ using a method described by Douglas and Jones (1963) (called implicit linearization). In one of the few studies which deal with comparisons of numerical simulation models, Haverkamp *et al.* (1977) found that the implicit methods used between 5 to 10 times less computer time than the explicit methods for the simulation of water infiltration in a sandy soil. Richter (1980) reported that the execution time could be substantially further reduced with a technique developed by Wind and Van Doorne (1975), which involved an integrated form of Darcy's law, based on an empirical

conductivity function, $K(\psi)$.

Although the analysis of water movement in soils has generally been limited to finite difference methods, such as the ones described above, more recently there has been considerable interest in Galerkin or equivalent finite element methods for numerical modeling of water flow in soils (Bruch and Zyvoloski, 1974; Neuman *et al.*, 1975; Yoon and Yeh, 1975; Hayhoe, 1978; Gureghian, 1981). The theoretical aspects of the finite element method, as applied to soil water flow problems have been discussed by Guymon (1974) and Neuman *et al.* (1975): basically the method is based on the approach of weighted residuals which leads to an integral representation of the problem.

Both the finite difference, and the finite element technique will yield similar results for simple, one-dimensional flow regimes (Feddes *et al.*, 1975; Pickens and Gillham, 1979). However it is generally acknowledged that the finite element technique is more ideally suited to deal with complex geometries, anisotropy and heterogeneity, which are characteristic of most practical soil water flow problems. Except for the study by Hayhoe (1978) there have been no comparisons among the two methods with regard to their relative efficiency.

Under laboratory conditions, where the accurate prediction of wetting and drying fronts is often a primary objective, the simulation runs commonly use compartment sizes of up to a few centimetres and variable time intervals of up to a few seconds (Hanks and Bowers, 1962; Staple, 1966; Shaykewich and Stroosnyder, 1977). Under field conditions, where the simulation runs are carried out to cover an entire growing season, and one is interested in more general water content profiles (and water uptake by plants) compartment sizes of up to 20 cm and time intervals varying from approximately 10 minutes to 2 hours have been used successfully (Nimah and Hanks, 1973; Feddes *et al.*, 1976; De Jong and Cameron, 1979).

C. The hydraulic properties of the soil

A solution of the physical based models can only be obtained if the water characteristic ($\psi(\theta)$) and the hydraulic conductivity function ($K(\theta)$ or $K(\psi)$) of the soil are known. Current methods of determining these two functions are time consuming, tedious and expensive. For this reason, in many instances, too few data points are obtained to define the total relationship with precision. Consequently, numerous attempts have been made to estimate the soil water characteristic curve from limited data (Husz, 1967; Rogowski, 1971; McQueen and Miller, 1974; Clapp and Hornberger, 1978; Gupta and Larson, 1979). Unfortunately, these models have not been tested extensively on a wide range of soils under field conditions where spatial variability is large. Moreover, the hysteretic effects in matric potentials, which affect water movement and storage (Staple, 1979) are not considered in these largely empirical models.

No fundamentally-based equation of general validity relates the hydraulic conductivity to the matric potential or the water content. Various empirical equations have been proposed (Gardner, 1958; Rytenga, 1965; Ahuja and Swartzendruber, 1972) but for individual soils, the

equation of best fit and the values of the parameters must be determined experimentally.

A more promising approach has been the prediction of the unsaturated hydraulic conductivity from pore size distribution data (Childs and Collis-George, 1950; Marshall, 1958; Millington and Quirk, 1961; Campbell, 1974). With some modifications and the inclusion of a matching factor (the ratio of the measured conductivity to the calculated one, at a given matric potential) the agreement between calculated and measured values has been satisfactory for coarse textured, apedal soils (Jackson *et al.*, 1965; Kunze *et al.*, 1968; Green and Corey, 1971; Mualem, 1976; Dane, 1980). However, the results for well structured, fine textured soils have been unreliable partly due to the presence of swelling and shrinking of cracks (Denning *et al.*, 1974; Fluhler *et al.*, 1976). Also the effects of biochannels (worm holes, crop root channels, etc.) which vary with time and depth in a rather random fashion have not been incorporated in these models. Moreover, there is, as yet, no discernible pattern to the matching factors, so one is totally in the dark in the absence of a known conductivity value. This of course seriously limits the usefulness of the physical based models.

D. The boundary conditions

The upper boundary condition of the physical based models is either rainfall and/or evaporation. Whereas under laboratory conditions the time and rate of water application is carefully controlled, under field conditions the time of the start and cessation of the rainfall, as well as its intensity is not a standard meteorological measurement. The rate at which rain falls is therefore calculated by arbitrarily assuming that the recorded total for the day fell at a constant rate during a certain period of the day (e.g. between 19.00 and 24.00 o'clock).

The rate of evaporation from a sufficiently wet soil surface, the energy limited rate, q_1 , can be estimated from the daily potential evaporation rate, which has been assumed to follow a half sine wave (Rowse and Stone, 1978) or a normalized curve (De Jong and Cameron, 1979) during a proportion of the day (e.g. during daylight hours). Frequently the rate of evaporation is much less than this, as it is limited by the rate at which water can be conducted to the soil surface. This is the soil limited rate, q_2 , which is calculated as the rate of upward transmission of water through the profile, after the upmost compartment has dropped to a given minimum matric potential or water content which is in equilibrium with the humidity of the ambient air (Hanks and Gardner, 1965; Hanks *et al.*, 1969). The actual flux at the soil surface is then calculated as the smaller of q_1 and q_2 .

Staple (1971, 1972) revised the upper boundary condition by expressing the evaporative flux in terms of windspeed, air temperature and humidity. Further revisions were made (Staple, 1974) by using a modified Penman equation (Penman, 1948, 1956), by including in it the relative vapour pressure of a partially dried surface soil, to estimate the surface flux. This, in turn, has led to the development of a numerical non-isothermal evaporation model (Van Bavel and Hillel, 1975, 1976) which solves the surface energy balance for the evaporative heat flux.

A variety of lower boundary conditions have been used in the physical based models. The presence of an impermeable layer is usually simulated by setting the flux at the lower boundary equal to zero for all time intervals (Hillel, 1977). Free drainage is accomplished by setting this flux equal to the hydraulic conductivity of the lowest compartment (Hillel and Van Bavel, 1976). The presence of a water table has been approximated by keeping the lower compartments at a constant (saturated) water content or at a given matric potential (Dutt *et al.*, 1972; Rowse, 1975; Van der Ploeg *et al.*, 1978). This latter condition permits both downward and upward flow at the basal boundary. De Laat *et al.*, (1975) use as basal boundary a shallow water table which fluctuates in accordance to capillary rise, percolation and horizontal groundwater movement.

E. Water extraction by roots

Water flow to plant roots has been studied by a number of investigators. Some studies (Philip, 1957; Gardner, 1960; Cowan, 1965; Molz *et al.*, 1968) concern the radial flow of water to a single root (microscopic models). For example, Gardner (1960) showed that this may be approximated by the steady solution of the equation of diffusion between two concentric cylinders:

$$\psi_s - \psi_r = (q/4\pi K) \ln (r_1^2 / r_2^2) \quad (16)$$

where ψ_s and ψ_r are the matric potential in the bulk soil and at the root surface respectively, q is the uptake rate per unit length of root, K is the hydraulic conductivity and r_1 and r_2 are the radii of the outer cylinder (equated to half of the main distance between roots) and the inner cylinder (equated to the root radius).

The detailed microscopic models do not lend themselves to interpretation of field data, gathered under heterogeneous conditions of soil water and over long periods of shoot and root development. Furthermore, different parts of this root-soil system interact with one another through vertical xylem potential gradients. Macroscopic models (Gardner, 1964; Whisler *et al.*, 1968; Molz and Remson, 1971; Feddes and Rytema, 1972) deal with the removal of water by the root zone as a whole without considering explicitly the effects of individual roots. These models are very general and the physiological parameters are usually so poorly specified that their use in interpreting field data or in predicting crop water use has been limited.

Recently several researchers (Nimah and Hanks, 1973; Hansen, 1975; Rose *et al.*, 1976; Hillel *et al.*, 1976; Van Bavel and Ahmed, 1976; Taylor and Klepper, 1978; Federer, 1979) have modeled soil water extraction by root systems using a somewhat "hybrid" approach, i.e. by incorporating several time specific plant and soil parameters into a calculation system so that various parts and coefficients interact at any one time and change with the passage of time. These systems predict the time course of transpiration given certain initial conditions for a particular crop and the time course of meteorological or plant conditions. The extraction of water by plant roots is then represented by a sink term (S) in the general soil water flow equation (3), i.e.:

$$\partial\theta / \partial t = \nabla \cdot \{K(\theta) \nabla\phi\} - S \quad (17)$$

Nimah and Hanks (1973) assumed that water loss from the crop equal either to a potential rate or equal to a rate at which water could be extracted by the root-soil system when the plant water potential reached a minimum specified value. The root extraction per unit depth (S) was calculated from the difference between root and soil potentials multiplied by the hydraulic conductivity of the soil and a root distribution function, estimated from measurements of root weight. However the calculation of S involved the improbable assumption that the resistance to water absorption into the root was negligible compared to an assumed resistance to water conduction along the root, and the distance between roots was always 1 cm.

The Nimah and Hanks model was further developed by Feddes *et al.* (1974); their main improvement was to calculate the root extraction term from:

$$S = K (\psi_s - \psi_r) / b \quad (18)$$

where S is the volumetric rate of water uptake per unit volume of soil, K , ψ_s , and ψ_r were defined previously, and b is an empirical proportionality constant which varies with depth and time to account for the variation of root density. Feddes *et al.* (1974) made the assumption that the potential at the root surface, (ψ_r) is constant throughout the root system. This assumption will only be valid if the root resistance is always small compared to that in the soil and evidence presented by Newman (1969a and b), Lawlor (1972) and Stewart *et al.* (1981) suggests that this is not always the case. Subsequent publications by Neuman *et al.* (1975) and Feddes *et al.* (1976) concern the numerical solution, but the fundamental assumptions about water extraction remained unchanged.

The models of Hansen (1975) and Hillel *et al.* (1976) include plant resistances and therefore do not have to make the improbable assumption of a constant potential at the root surface. Essentially both models assume that uptake at any depth is given by:

$$S = (\psi_s - \psi_p) / (R_s + R_p) \quad (19)$$

where ψ_s and ψ_p are the soil and plant potentials and R_s and R_p are the corresponding resistances. Uptake from all layers is equated to transpiration. Hansen (1975) uses an expression based on equation (16) to calculate the soil resistance, while Hillel *et al.* (1976) use an expression similar to equation (18).

Hansen (1975) calculates the plant resistance for each soil depth from the length of a root at a given depth and the resistance per unit length of root which was determined by dividing the measured total plant resistance by the total root length. Hillel *et al.* (1976) consider the plant resistance in two parts, an absorption or cortical resistance and a conduction or xylem resistance. The conduction resistance was assumed to be proportional to the distance of the root from the soil surface, which implies that the water pathways from each depth do not mix until they reach a point in the plant near the soil surface (the "crown"). In general, the conduction resistance is thought to be small although there is some evidence to the contrary (Passioura, 1972; So, 1979).

Taylor and Klepper (1978) also sum uptake from horizontal layers, but their model is limited to water movement from bulk soil to, into and up the root xylem to the "crown". Their model varies from that of Hillel *et al.* (1976) in two significant ways. First Taylor and Klepper use hourly values of "crown" water potential as an input variable and second they consider that conduction resistance is a function of both depth and flow rate, while Hillel *et al.* (1976) specify that this is only a function of depth.

The root length density - or root weight function, which is a required input variable to most of the models described varies with physiological factors like species, growth stage and photosynthate supply to the roots, as well as with edaphological factors like soil water supply, soil temperature, fertility status, etc. The experimental evaluation of such a function is therefore difficult and time consuming (Böhm *et al.*, 1977) and it is tempting to assume that the root length density varies with depth in some mathematically convenient way (Feddes *et al.*, 1974; Van Bavel and Ahmed, 1976). One can also model root length densities from root initiation and extension rates (Hackett and Rose, 1972; Lungley, 1973) and death rates (Hillel and Talpaz, 1976) or by analogy from the equations describing diffusion (Page and Gerwitz, 1974; Hayhoe, 1979). No attempts have been made to make the root growth function dependent upon the results of the soil water model itself.

F. Comparison of the physical based models

In the physical based models the equation of continuity and Darcy's law are combined to form the general soil water flow equation. Most investigators have formulated this equation such that the hydraulic head gradient forms the driving force (Table 1). However, by using the diffusivity form of the equation, it has also been formulated in terms of a water content gradient. Shaykewich and Strooisnder (1977) used the matric flux potential gradient as the driving force. Whether the equation is then solved in terms of water contents or pressure heads is irrelevant since with the use of the soil water characteristic one can easily convert one into the other.

Due to highly variable boundary conditions and changing soil properties throughout the profile, the solution of the differential equations governing flow, is usually achieved by numerical methods. Since the finite difference techniques are easier to program as compared to the finite element techniques, the former ones have been used more extensively (Table 1). However, because the finite element techniques are more ideally suited to deal with complex geometries they have recently received more attention. Explicit or implicit methods are both used frequently although it is worth nothing that all finite element techniques use the implicit method of solving a system of linearized equations. The relative advantages of each technique and method have been discussed by Guymon (1974) and Haverkamp *et al.* (1977).

Table 1. A comparison of various physical based soil water models.

Author	Date	Driving force			Form of solution			Method of solution			Boundary conditions		Root extraction term		Soil	Dimension			Hysteresis										
											(upper)	(lower)																	
		hydraulic head grad.	water content grad.	matric flux potential grad.	water content profile	pressure head profile	finite difference	finite element	explicit	implicit	precipitation	evapotranspiration	surface energy balance	free drainage	impermeable layer	quasi—dynamic watertable	dynamic watertable	macroscopic approach	hybrid approach	plant resistances included	homogeneous	layered	1D	2D	3D	single valued function	hysteresis		
Hanks <i>et al.</i>	1969	X			X		X			X	X		X	X	X			X			X					X		X	
Freeze	1971	X				X	X			X	X		X				X				X						X		X
Holz and Remson	1971		X		X		X			X	X		X		X			X			X					X			X
Staple	1971	X			X		X	X			X	X	X	X				X			X						X		X
Van Keulen and Van Beek	1971	X			X		X	X			X	X		X	X			X			X					X			X
Dutt <i>et al.</i>	1972		X		X		X	X			X	X	X	X				X			X					X			X
Nimah and Hanks	1973	X			X	X	X		X		X	X	X	X		X		X	X		X					X			X
Feddes <i>et al.</i>	1974	X			X		X		X		X	X	X	X		X		X			X					X			X
Hansen	1975	X			X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
Neuman <i>et al.</i>	1975	X			X	X	X		X	X	X	X	X	X		X		X	X	X	X		X			X			X
Yoon and Yeh	1975	X			X		X	X	X		X	X	X	X	X		X		X	X	X					X			X
Hillel <i>et al.</i>	1976	X			X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
Hillel and Van Havel	1976	X			X		X	X	X		X	X	X	X	X		X		X	X	X					X			X
Van Ravel and Hillel	1976	X			X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
Feddes and Zaradny	1977	X			X		X		X		X	X	X	X		X		X	X	X	X					X			X
Shaykewich and Strooisnyder	1977		X		X		A		X		X	X	X	X		X		X	X	X	X					X			X
Hayhoe	1977		X		X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
Rowse <i>et al.</i>	1978		X		X		X	X	X		X	X	X	X	X		X		X	X	X					X			X
Van der Ploeg <i>et al.</i>	1978	X			X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
De Jong and Cameron	1979		X		X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
Federer	1979	X			X		X	X	X		X	X	X	X		X		X	X	X	X					X			X
Gureghian	1981	X			X		X	X	X		X	X	X	X		X		X	X	X	X					X			X

Most of the papers cited in Table 1 accept as upper boundary condition precipitation and evapotranspiration data. Some are capable of solving a surface energy balance, using atmospheric data as input. The lower boundary condition involves free drainage, an impermeable layer, a quasi dynamic or a dynamic water table. The quasi dynamic boundary condition involves a fluctuating water table (i.e. a pressure head equal to zero at a given depth), but whose fluctuations are predetermined by the authors. A true dynamic watertable behaviour such as reported by Freeze (1971) and Gureghian (1981) is one that is the result of an integrated solution involving both saturated and unsaturated zones.

Many authors have treated the root extraction term rather empirically (macroscopic approach) due to our lack of understanding of soil water flow into the roots and through the plant. Those authors which incorporate time dependent soil- and plant parameters (like resistance terms) use the so called hybrid approach.

The papers cited in Table 1 treat the soil either as homogeneous or layered. Most laboratory studies have dealt with homogeneous soils, while field studies often involve a layered soil profile. Most authors treat the case of one dimensional vertical flow only, with a single valued functional relationship between water content and pressure head.

The choice of available physical based soil water models is large. Each model has its own particular advantages and disadvantages in terms of basic mathematical formulation, method of solution, boundary conditions and root extraction terms. None of them is entirely satisfactory for all possible applications, but one that is adaptable to almost every problem can be found in the literature.

III. The soil water budget models

A. General description

Since long term data are available for numerous weather stations in important agricultural areas, several methods have been suggested for compiling daily water balances from standard meteorological observations. Most of these methods make use of the concept of potential evapotranspiration (PE) as an indicator of the possible maximum loss of water from the soil under existing atmospheric conditions. Thornthwaite (1953) suggested a simple bookkeeping scheme for scheduling time and rate of irrigation based on daily estimates of PE. Starting with a known soil water content, the daily values of PE are subtracted from the daily rainfall and the result is subtracted (or added when the rainfall exceeds PE) from the water present in the soil to give the new water storage. The process is repeated on a daily basis. When the water content of the soil profile reaches field capacity¹, e.g. in the event of heavy rains, the excess precipitation is assumed to be lost from the profile by deep drainage. When it falls to the permanent wilting point² no further water is extracted from the profile. Under irrigation an amount of water equal to the deficit between the water content at a predetermined level and field capacity is applied.

At this point it is desirable to introduce the concepts of water use by plants as employed in the budget models. The rate at which plants use water has been a subject of debate for many years. Several reviews have dealt with this controversy (Veihmeyer and Hendrickson, 1955; Baier, 1967).

Plants usually have little difficulty in removing water from soil which is at or near field capacity: the actual evapotranspiration rate (AE) can be assumed to equal the potential rate (PE). However, as the soil dries, the rate of water uptake by plants (or the actual evapotranspiration rate) decreases sharply. This decreasing water uptake rate depends not only on meteorological conditions, but also on the type of vegetation, the soil type and the soil water content (Pierce, 1958; Lowry, 1959; Denmead and Shaw, 1962; Holmes and Robertson, 1963). The soil water budget models employ empirical relationships expressing the ratio AE to PE as a function of the amount of water remaining in the soil. The unknown degree of shifting and tilting of these drying curves, in response to meteorological soil and plant factors, restricts the usefulness of the budget models.

In contrast to the so called "single" soil water budget models which do not account for soil water stress changes by zones or expanding root systems (Van Bavel, 1955; Thornthwaite and Mather, 1955; Marlatt *et al.*, 1961; Fitzpatrick and Nix, 1969) the "modulated" budget models divide the soil into zones and budget the actual evapotranspiration to those zones (Holmes and Robertson, 1959; Shaw, 1963; Shaw, 1964). Results presented by Holmes (1961)

¹ Field capacity is defined as the percentage of water remaining in the soil 2 or 3 days after the soil has been saturated and free drainage has practically ceased.

² Permanent wilting point is defined as the water content of a soil at which plants, specifically sunflower plants, wilt and fail to recover their turgidity when placed in a dark humid atmosphere.

indicate that "modulated" budget models are a considerable improvement over "single" budget models.

Most existing soil water budgeting techniques are designed for specific purposes and are not universally adaptable. A new technique for the estimation of daily soil water on a zone by zone basis from standard climatological data was presented by Baier and Robertson (1966). This method, which is more versatile than previously published ones, makes use of some basic concepts of earlier budget models, such as taking PE as a possible maximum of AE and subdividing the total available soil water into several zones of different capacities. This model also facilitates water withdrawal simultaneously from different depths of the soil profile permeated by roots in relation to the rate of PE and the available soil water in each zone. Adjustments for runoff, deep drainage, different types of soil drying curves and the effect of different atmospheric demand rates on the AE/PE ratio are also incorporated.

Based on experience, user's comments and subsequent research (Baier and Robertson, 1967, 1968; Baier 1969a, b) detailed instructions of this "Versatile Budget" were described and listed in a technical bulletin (Baier *et al.*, 1972). In 1979 this bulletin was updated, with some minor modifications (Baier *et al.*, 1979).

While many of these budget models are sufficient for the purposes for which they were developed, one complaint is that the movement of soil water (except for the recharge to field capacity during rainfall) is ignored. While this may be satisfactory for irrigation scheduling, it may be detrimental to studies such as soluble salt distribution (De Jong, 1974). Furthermore prairie soils are usually below field capacity for long periods and water changes by unsaturated flow could be important. Also the budget models are not capable to account for topographic effects, i.e. horizontal soil water movement.

The above described soil water budget models do not account for the upward transfer of water from layers underlying the "modeled" soil, and thus cannot be expected to simulate water in soils above a shallow water table. Baier and Robertson (1966) recommend that their model be used only after July 1 in the eastern Canadian wheat zone, and Shaw (1963) found the poorest results of his model in southeast Iowa on soils having shallow water tables. Modifications of this lower boundary condition will be described in one of the next sections.

B. The concepts of field capacity and permanent wilting point

Early observations that the rate of flow and water content changes decrease in time (Alway and McDole, 1917; Veihmeyer and Hendrickson, 1931) have been interpreted to indicate that the flow rate becomes negligible within a few days. The presumed water content at which internal drainage allegedly ceases, termed field capacity, has for a long time been accepted as an actual physical property, characteristic of, and constant for each soil.

However, in recent years, with the development of theory and more precise experimental techniques in the study of unsaturated flow processes, the field capacity concept has been recognized as arbitrary and not an intrinsic physical property independent of the way it is measured (Richards, 1960). The redistribution process is in fact continuous and exhibits no abrupt "breaks" or static levels. Although its rate decreases constantly, in the absence of a water table the process continues and equilibrium is approached, if at all, only after very long periods.

The soils for which the field capacity concept is most tenable are the coarse textured ones, in which the hydraulic conductivity drops most steeply with decreasing matric potentials and flow becomes slow relatively soon. However, in the medium and fine textured soils, redistribution can persist at an appreciable rate for many days (see e.g. Wilcox, 1959; Miller and Aarstad, 1974), although when cracks and biochannels are present the hydraulic conductivity also drops very steeply with decreasing matric potentials (Keng and Lin, 1980).

The rate of outflow from any given layer in the soil depends not only on the texture or hydraulic characteristics of that layer, but also on the composition and structure of the entire profile, since the presence at any depth of an impeding layer can retard the movement of water out of the layers above it. Thus, the storage capacity of the soil is related not only to time, but also to the textural composition and layering sequence of the profile, as well as to its initial water-content distribution.

Despite all these objections, the field capacity concept is still considered by many to be a useful criterion for the upper limit of plant available water. As such it should be measured in the field (Rickard and Cossens, 1967) since there is no generally satisfactory laboratory procedure for obtaining it. The values of the various proposed laboratory methods depend on the degree to which their results correlate with field measurements. Matric potential values such as $-1/3$, $-1/10$ or $-1/20$ atmosphere may represent a measurable field capacity in certain circumstances (Colman, 1947; Haise *et al.*, 1955; Salter and Williams, 1965; Webster and Beckett, 1972), but it is fundamentally wrong to assume that such criteria will hold universally since they are static in nature, while the redistribution process is essentially dynamic.

The permanent wilting point has been accepted as the lower limit of available water. Different workers have used different degrees of wilting to determine the direct wilting percentage with sunflowers (Briggs and Shantz, 1912; Taylor *et al.*, 1934; Work and Lewis, 1934). With sufficient replication the wilting percentage obtained by any one group of workers is reproducible, but it varies with the different wilting symptoms used. Furr and Reeve (1945) found a wide range in water content between what they called the first permanent wilting point, which is indicated by the wilting of the lower leaves of sunflower plants, and the ultimate wilting percentage, indicated by the wilting of top leaves. They showed that the wilting range between those two extremes comprised 11 to 30% of the total available water.

Richards and Weaver (1943) studied the matric potential with which soil water was held at the permanent wilting point. They concluded that on the average the water retained at -15 atmosphere was the best estimate. Since then a number of relationships between the permanent wilting point and the 15 atmosphere percentage have been determined (Richards and Wadleigh, 1952; Haise *et al.*, 1955; Wilcox, 1960; Lehane and Staple, 1960). These relationships are all very similar if the same wilting criteria are used.

Although originally presented as a sole soil property, it is now generally accepted that plant properties and environmental conditions also influence the permanent wilting point. For example, Staple and Lehane (1941), Rennie and Huchon (1961) and Campbell *et al.* (1977) found that wheat, grown under dry land conditions in Saskatchewan can reduce the water content of the soil significantly below the wilting point.

Despite the criticism against the concepts of field capacity and permanent wilting point, they still receive considerable attention because -

- a) these 'properties' are easier to measure, or estimate than soil water retention curves and hydraulic conductivity functions;
- b) the concept of field capacity might be used to solve the problems of water flow associated with cracks and biochannels (Rowse, 1975);
- c) in the region where wilting occurs, the relationship between water content and matric potential is highly non linear.

C. The boundary conditions

The soil water budget models accept rainfall and/or evaporation as an upper boundary condition. As the budget models are run on a daily, or occasionally on a weekly (Fitzpatrick and Nix, 1969) basis, and the meteorological measurements are on a daily basis, there is no need in making assumptions regarding the time of start and cessation of rainfall or evaporation and the rate at which these processes occur.

Baier *et al.* (1972, 1979) have included a snow budget and snow melt subroutine in their budget model for winter conditions in Canada. In the snow budget, precipitation (snow) is accumulated on the ground when the smoothed daily maximum air temperature does not exceed an empirically developed threshold value. An empirical "snow coefficient", related to ground cover, accounts for the proportion of snow which blows away and/or evaporates. Snowmelt terms account for the contribution to soil water from the snowpack. The factors considered are the rate of snowmelt, the retention of meltwater within the snowpack and the rate of penetration of meltwater into frozen or partially thawed soil.

In most soil water budget models evaporation from bare soil is related to PE and a soil factor, which limits the potential evaporation rate. This soil factor is represented by an empirical drying curve which relates the ratio AE/PE to the amount of available water left in the soil. Used

as such, the drying curve represents the hydraulic properties ($\psi(\theta)$ and $K(\theta)$) of the soil. Baier and Robertson (1966) used such a drying curve and empirical coefficients which decreased from the upper zone downward. In this case the drying curve represents the soil water characteristic and the coefficients represent the hydraulic conductivity function, although direct relationships among them have never been established. The coefficients, which were estimated to resemble the most probable extraction pattern from bare soil under given prevailing environmental and soil conditions are not necessarily transferable to different conditions. This was shown by Ravelo (1978) who developed different coefficients for bare soil evaporation for climatological and soil conditions in Missouri.

The lower boundary condition of the soil water budget models allows deep drainage during a one day time step, until the lowest zone reaches field capacity. If the water content of this zone is lower than field capacity, no water moves into or out of the modeled soil profile. Exceptions are the models developed by Makkink and Van Heemst (1975) and Stuff and Dale (1978) which include estimates of the capillary contribution from a shallow water table. De Jong and Shaykewich (1981) modified the lower boundary condition to account for nearly impermeable layers as found in poorly or imperfectly drained soils. The main objection against these improved lower boundary conditions is, that considerably more knowledge of the physical soil properties is required.

D. Water extraction by roots

Water extraction by plant roots in the budget models is handled very similar to evaporation from bare soil, except that besides PE and a soil factor, a plant factor is also included to limit potential transpiration to actual transpiration. This plant factor might be a root density function (see e.g. Shaw, 1963; De Jong and Shaykewich, 1981) which can change with depth and time, or a single crop coefficient (Jensen *et al.*, 1971 Aase *et al.*, 1973) which changes with time, or a set of crop coefficients (Baier and Robertson, 1966) which change with depth and time. The plant factor is crop specific, although certain similar crops are usually grouped together. Single crop coefficients such as developed by Blaney and Harris, (1951), Denmead and Shaw, (1959), Erie *et al.*, (1965) are generally used in "single" soil water budget models, whereas the multiple crop coefficients are used in "modulated" budgets.

The major limitation of the soil water extraction terms in budget models is the empirical nature of the soil and plant factors. Extreme caution has to be exercised upon transferring these factors from one climatological and soil environment to the next.

E. Comparison of the soil water budget models.

In the budget models water movement in the soil is treated according to the field capacity concept: during rainfall or irrigation the soil is wetted to field capacity. Any additional rainfall, after the whole soil profile has reached field capacity, is considered to cause deep drainage or

runoff. Subsequent redistribution of water within the profile, or capillary use from underlying water table is not accounted for.

The soil water budget models have been formulated either in terms of single layered ones, or multiple layered or modulated ones (Table 2). The latter ones will give better estimates of the distribution of soil water within the root zone, but not necessarily better estimates of the total water content of the soil profile.

Most soil water budget models operate on a daily basis, except the ones developed by Fitzpatrick and Nix (1969) and Aase *et al.* (1973) who use weekly time steps. While this might cut down on computational costs, it is less convenient in that the input data is usually available on a daily, rather than a weekly basis. Moreover the occurrence of rainfall and evapotranspiration within any single week is variable and dissimilar temporal patterns of surface wetting and drying may result.

Field capacity and permanent wilting point data, or the difference between them, called plant available water, are required input data to all soil water budget models (Table 2). Unfortunately, the field capacity concept ignores, except for the recharge during rain-fall, the movement of soil water. While this might be of little consequence in coarse textured soils, in medium and fine textured soils the redistribution process persists at appreciable rates for many days. Also the use of a specific matric potential to approximate field capacity must be viewed with caution: one deals with dynamic processes, as opposed to a static one, as implied by a single matric potential value for field capacity. The permanent wilting point has been accepted as the lower limit of available water. A number of relationships have been proposed to relate the permanent wilting point to the 15 atmosphere percentage. While these relationships are all very similar, exceptions have been noted, because beside soil factors, plant factors also determine the wilting point.

All the papers cited in Table 2 accept as upper boundary condition rainfall and evapotranspiration data. The models developed by Makkink and Van Heemst (1975) and Baier *et al.* (1979) also deal with snowfall and snowmelt and consequently can be run overwinter. The lower boundary condition of the budget models does not allow for upward transfer of water from shallow watertables, except the ones developed by Makkink and Van Heemst (1975) and Stuff and Dale (1978). De Jong and Shaykewich (1981) modified the lower boundary condition to account for impermeable layers as found in poorly drained soils.

Water use by plants is proportional to potential evapotranspiration, a soil factor and a plant factor, except in the model developed by Marlatt *et al.* (1961) where water use is independent of a crop factor. The soil factor is always represented by an empirical drying curve, relating the ratio AE/PE to the amount of available water left in the soil. The plant factor can include an empirical single - or multiple crop coefficient or a root density function.

Many of the functional relationships employed in the soil water budget models are empirical. The choice then, of which one is most useful, should be based upon whether or not a particular model has been calibrated for the specific soil, crop and climatic conditions for which it is intended to be used.

Table 2. A comparison of various soil water budget models.

Author	Date	Formulation	Time Step	Soil's input			Boundary Conditions			Water use by plants		
							(upper)		(lower)			
		single modulated	1 day 1 week	field capacity permanent wilting point plant available water								
Holmes and Robertson	1959	X	X	X	X	X	X	X	X	X	X	X
Marlatt <i>et al.</i>	1961	X	X	X	X	X	X	X	X	X	X	X
Shaw	1963	X	X	X	X	X	X	X	X	X	X	X
Baier and Robertson	1966	X	X	X	X	X	X	X	X	X	X	X
Fitzpatrick and Nix	1969	X	X	X	X	X	X	X	X	X	X	X
Jensen <i>et al.</i>	1971	X	X	X	X	X	X	X	X	X	X	X
Aase <i>et al.</i>	1973	X	X	X	X	X	X	X	X	X	X	X
Makkink and Van Heemst	1975	X	X	X	X	X	X	X	X	X	X	X
Stuff and Dale	1978	X	X	X	X	X	X	X	X	X	X	X
Baier <i>et al.</i>	1979	X	X	X	X	X	X	X	X	X	X	X
De Jong and Shaykewich	1981	X	X	X	X	X	X	X	X	X	X	X

IV. Combination models

A. Introduction

Both the physical based, and the water budget models have shortcomings: the physical based models require considerable knowledge of the processes which control water storage and movement. Consequently detailed information on the soil, plant and atmospheric characteristics, which influence these processes, is required. Unfortunately, at the present time it is generally acknowledged that the processes, which control water uptake by roots have not been elucidated completely, and the data base for the appropriate dynamic soil-and plant properties is very limited.

The budget models are largely based on soil properties whose meaning has been questioned seriously in recent years. Moreover, many of the functions in the budget models are empirical, and therefore not necessarily transferrable to different climatological and soil's conditions, other than those for which they were derived.

In view of these limitations of both the physical based-and the water budget models, several attempts have been made recently to combine both types of models into what can be called "combination" models or modified models.

B. Some examples

It has been shown (Van Keulen and Van Beek, 1971) that the maximum permissible time interval that can be used without oscillations occurring in the physical based models is a function of the diffusivity ($D(\theta)$). The high values of diffusivity that are typically found between saturation and field capacity mean that very small time intervals have to be used during infiltration. This inevitably means a large increase in computer running time. To overcome these problems, several researchers (Rowse, 1975; Lambert *et al.*, 1976) who employed physical based models, made the assumption that water entering the soil progressively wets each layer to field capacity. In a subsequent paper, Rowse and Stone (1978) reported that this method seriously over-estimated the amount of water retained in the surface layers due to the presence of cracks and/or hysteresis. Therefore they used the following empirical method to distribute rainfall in the soil profile: at the start of rainfall each soil layer is considered to have a soil water deficit equal to the difference between its actual water content and that at field capacity. During rainfall each layer fills progressively from the soil surface so that in each layer only half of this deficit is replaced.

Despite the fact that progress is being made in the elucidation of facto which control water uptake by plants, there is as yet no complete physical based model which can predict the time course of transpiration for various soil, crop and climatic conditions. Consequently the sink term in the soil water flow equation (see eqn. 17) has been approximated by "borrowing"

concepts originally developed for budget models. For example, Dutt *et al.* (1972) calculate the water extraction from the entire profile from a consumptive use factor (a crop coefficient) and the climatic factors temperature and day length. This total water extraction is then distributed throughout the profile in proportion to a constant extraction pattern, such as the average root distribution. De Jong and Cameron (1979) also use a root density function to distribute the total potential transpiration over the entire profile. At each zone, water extraction was then limited through the use of an empirical drying curve.

In the field the root density function varies with the type of soil and the rooting system, which usually changes with depth and time. Experimental evaluation of such a function is time-consuming and costly, and mathematical models which describe root growth have not yet been fully evaluated under field conditions. For these reasons Feddes *et al.* (1976) proposed to use a root extraction term that depends on the soil water content, the potential transpiration rate and on the depth of the root zone, which is easier to estimate than a root density function. To describe this function the following assumptions were made. For conditions drier than the wilting point (θ_3) and wetter than a certain anaerobiosis point (θ_1) water uptake by roots is assumed to be zero. Uptake is taken to be maximal when the soil water content is between θ_1 and θ_2 , a value corresponding with a matric potential at which soil water begins to limit plant growth. A linear decrease in uptake occurs between θ_2 and θ_3 . With this simple model, which is basically an extension of the drying curve concept used in budget models, Feddes *et al.* (1976) and Van der Ploeg and Beese (1977) obtained rather satisfactory results.

Further investigations by Feddes and Zaradny (1977) showed that the model can be improved by considering the sink term depending on the matric potential ψ , instead of on the water content θ . This complies with those types of models that consider the sink term to be proportional to the difference in matric potential between the soil and the root interior.

There is ample evidence (Wilcox, 1959) (Gardner *et al.*, 1970) that redistribution of water in soil following infiltration, in the absence of a water table, is a continuous process. The importance of this is, that it determines the amount of water held near the surface for subsequent use by plants (Wilcox, 1962). Hastie (1976), recognizing the deficiencies of soil water budget models with regard to redistribution, incorporated Darcy's law into the budget model of Baier and co-workers to account for vertical redistribution of soil water within the profile. Unfortunately, the redistribution sub-routine failed due to stability problems. More recently drainage- as well as redistribution coefficients have been incorporated into the soil water budget models (Broughton and Foroud, 1978; Dyer and Baier, 1979) in order to simulate soil water conditions in the range between saturation and field capacity and to predict changes in watertable depths. Although these coefficients are empirical, presumably they can be related to the hydraulic conductivity ($K(\theta)$) and diffusivity ($D(\theta)$) function.

The modified lower boundary condition of soil water budget models such as described in section III, C, are really combinations of physical-based and soil-water budget models.

The model developed by De Laat (1980), which describes unsaturated flow above a shallow water table, can also be viewed as a combination model. The unsaturated zone was schematized into two homogeneous layers; the effective root zone, which is the top layer where most of the roots are present and the subsoil with a variable phreatic surface as the lower boundary. The effective root zone is treated very much like a soil water budget model: unsaturated flow within this zone is neglected. Actual evapotranspiration is calculated from soil water conditions in the root zone and potential evapotranspiration. However water movement in the subsoil is treated according to physical based principles: transient vertical flow is simulated by a sequence of steady state situations, subject to boundary flux conditions at the soil surface (rainfall or evapotranspiration) and from below the water table. At this lower boundary condition the model is linked to a two-dimensional horizontal saturated flow model.

Most hydrologic models, which are primarily concerned with surface runoff and groundwater flow, can also be classified as combination models. For example, Holtan et al. (1975) and Whiteley and Ghate (1978) employ the concepts of field capacity and permanent wilting point to calculate evapotranspiration and allow no water movement when the soil water content is below field capacity. However, between saturation and field capacity downward seepage is computed as being proportional to the hydraulic conductivity and the water content of the layer under consideration. An empirically derived infiltration equation, which includes a saturation deficit term, describes water movement through the soil surface. Runoff occurs whenever rainfall exceeds the rate of infiltration.

The hydrologic models deal with soil water phenomena on the scale of a complete watershed or drainage basin. Although weather conditions are assumed to be uniform over the area under consideration, spatial variability is accounted for through the use of uniform hydrologic response areas. These internally homogeneous areas have been based on soil type, topographic and land use characteristics, as well as on combinations of them (England and Onstad, 1968; England and Holtan, 1969; McGuinness and Edwards, 1975). Excess rainfall-and subsurface saturated flow routing across each of these areas has preoccupied many watershed modelers, at the expense of modeling water flow and uptake by roots in the unsaturated soil zone.

C. Comparison of the combination models

Models which combine certain elements of both physical based-and budget models are termed combination models. In general terms, the combination models are not as rigorously formulated as the physical based models, but on the other hand they contain less empirical approximations than found in the budget models.

A comparison of the combination models (Table 3) reveals that most of them are derived from either physical based or soil water budget models. Watershed models, such as developed by Holtan et al. (1975) and Whiteley and Ghate (1978), deal primarily with overland flow and groundwater recharge in order to forecast stream flow. Unsaturated flow and water use by plants have been given very little attention in these models.

In those models where the original concept was based on physical principles, either the root extraction term has been simplified, or water movement during rainfall has been treated according to the field capacity concept (Table 3). Modification of the soil water budget models have involved redistribution of water, both upward and downward, during periods with no rainfall.

The model developed by De Laat (1980) can be viewed as a budget model (the root zone) overlying a physical based model (the subsoil plus the saturated zone underneath), with the two linked through the flux at the interface. A steady state situation is applied to the physical based part of the model.

Table 3. A comparison of various "combination" models.

Author	Date	Original concept			Main modification		
		physical based model	soil water budget model	watershed model	water extraction by roots	(infiltration) redistribution	(drainage) of soil water
Dutt <i>et al.</i>	1972	X			X		
Holtan <i>et al.</i>	1975			X		X	X
Rowse	1975	X				X	X
Lambert <i>et al.</i>	1976	X				X	X
Feddes and Zaradny	1976	X			X		
Hastie	1976		X			X	X
Broughton & Foroud	1978		X				X
Whiteley and Ghate	1978			X		X	X
De Jong and Cameron	1979	X			X		
De Laat	1980	X	X				X

V. Discussion and conclusions

The soil water models which have been published in the scientific literature are internally consistent and generally simulate soil water conditions and water uptake by roots well for the specific soil, climatic and crop conditions for which they were developed. The success of these models can partly be attributed to the fact that most of them use a calibration procedure to fit the observed soil water data. For example, De Jong and Cameron, 1979 modified the hydraulic properties of the soil, within the range of the observed data, to obtain a better fit. The extraction of water by plant roots usually contains an optimization parameter (e.g. the term b in eqn. 18, or the values of ψ_p and R_p in eqn. 19) which can be adjusted to obtain more meaningful results. The budget models have been used successfully by adjusting the root extraction coefficients (Baier and Robertson, 1966), or the values of field capacity and/or permanent wilting point (De Jong and MacDonald, 1975). However these optimization procedures should not be seen as serious limitations, because modeling and measurement programs should compliment each other, in order to successfully monitor soil water conditions on a large scale (Schmugge *et al.* 1981).

The level of input required to solve soil water models varies widely: the physical based models need detailed information on soil water characteristics, hydraulic conductivity functions and plant properties. While in principle the weather parameters are required at a frequency level corresponding to the time interval of the model, most physical based models can presently use daily weather information. The soil water budget models require considerably less information; field capacities and permanent wilting points are the requirements with regard to the soil information. Empirical crop characteristics, which have been established through statistical techniques, and daily or weekly weather data are also required to execute the budget models. The input level of the combination models is between the two described above: while they require similar soil information to that required by the physical based models, their need for detailed plant properties is much less stringent and in some cases approximates the input level required by the budget models.

All three types of models can use similar weather data bases. The principal requirements call for daily precipitation and potential evapotranspiration measurements. The problems of spatial variability of the weather parameters are equally applicable to all types of models, as well as estimating potential evapotranspiration from simple weather data.

Soil and crop data bases required to run the physical based models are very limited. While individual researchers may have specific data sets on which the physical based models can be run they certainly are not very extensive, in that they do not cover a wide range of soil and crop conditions. The required soil data base for the budget models is also very limited, although, when one third and fifteen atmosphere percentages can be related to field capacity and the permanent wilting, the data base is considerably larger. The crop data base for the budget models is fairly adequate: empirical crop characteristics, either in the form of single, or multiple crop coefficients, are now available for many economically important crops. However, since

these coefficients are empirical, caution has to be exercised upon transferring them to environments other than those for which they were derived.

Attempts to use the purely physical based models for estimating soil water conditions and crop water use for large groups of soils is not realistic at this time, because of a) the required high input level coupled with a limited soil and crop data base; b) our limited knowledge of water uptake by plant roots and the associated transpiration process. The combination models, attractive as they are, also require an input level which is generally not available. However, by expanding the soil and crop data base in terms of field measurements of soil water characteristics, hydraulic conductivity functions and rooting characteristics; modified versions of the physical based, and combination models will be suitable for estimating regional soil water conditions.

An adequate data base for the budget models can be created, if procedures can be developed which relate one-third and fifteen atmosphere percentages to field capacity and permanent wilting point (see e.g. Oosterveld and Chang, 1980). The budget model developed by Baier and co-workers may then be suitable to estimate soil water conditions and water use by crops for situations where unsaturated flow is negligible.

With the inclusion of a redistribution and/or water table subroutine, which may require redefining the empirical crop coefficients, the budget models can become more universally applicable. The inclusion of such a subroutine will require hydraulic properties as input. Since the data base of these properties is very limited, attempts have to be made to estimate them from basic soil constituents and the limited data available.

The inclusion of a single, crop specific, root-water extraction function in physical based - or budget models would be a further desirable development. For example root development could change not only in a predetermined fashion throughout the growing season, but also according to actual soil water profile conditions.

Most of the presently developed models are site specific. Spatial variability in terms of weather parameters, soil properties, land use and elevational changes are generally not incorporated into the models. If areal estimates of soil water conditions are required, it is generally assumed that the area under consideration is uniform in space, both above and below the soil surface, i.e. one extrapolates linearly from site specific conditions. To what extent this areal extrapolation can be carried out without jeopardizing the original site specific results will depend on the spatial variability of the above mentioned parameters.

The hydraulic models, which are unfortunately primarily concerned with streamflow generation and hydrograph forecasting, are an exception to the one-dimensional transfer models: they incorporate a certain degree of spatial variability through the use of uniform hydrologic response areas. This concept deserves further consideration in the soil water models which are mainly concerned with soil water conditions for agriculture and water use by crops.

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