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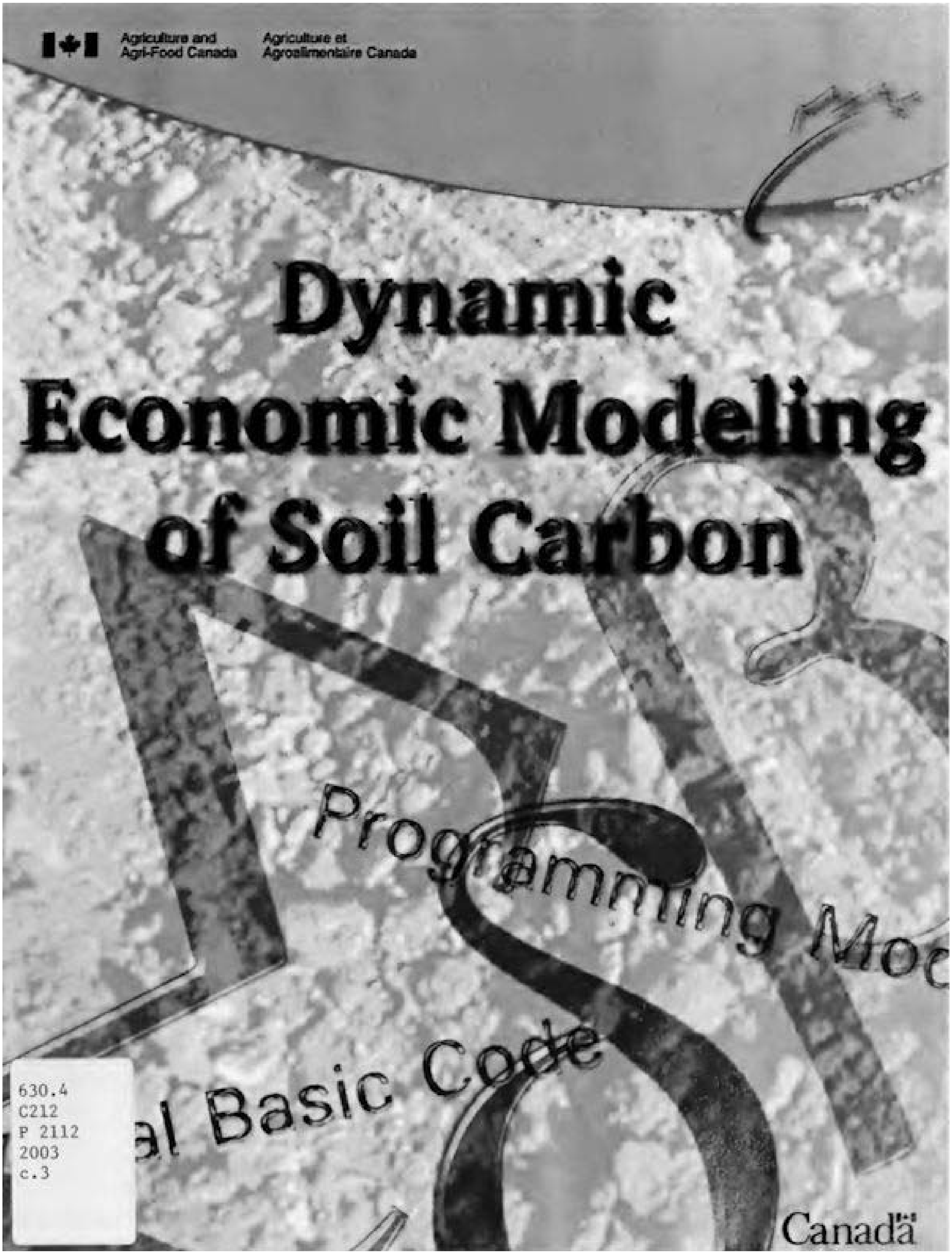
Dynamic Economic Modeling of Soil Carbon



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DYNAMIC ECONOMIC MODELING OF SOIL CARBON

Technical Report #1-02

Prepared by

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DYNAMIC ECONOMIC MODELING OF SOIL CARBON

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Table of Contents

List of Figures and Tables

Figures

Tables

Foreword

The Government of Canada and the provincial and territorial governments are working with the agriculture and agri-food industry and interested Canadians to develop an architecture for agricultural policy for the 21st century. The objective of the Agricultural Policy Framework (APF) is for Canada to become the world leader in food safety and food quality, innovation and environmentally-responsible production. To contribute to these goals, Agriculture and Agri-Food Canada (AAFC) has an ongoing research program to provide information on the effects of agricultural policy and technology scenarios on the environment and on the economic performance of the agriculture sector.

Included in this work program is the economic evaluation of various scenarios for the mitigation of greenhouse gas (GHG) emissions in the agriculture sector. Carbon sequestration practises in primary agriculture comprise one such mitigation scenario. This report analyses and models the potential decision-making process of farmers when they are presented with the opportunity to enter into carbon sequestration contracts. The focus of the report is on the complex factors and uncertainties involved in deciding whether and when to enter into such contracts. The report uses dynamic optimization methods to simulate potential solutions to the decision-making problem, and provides an analytical framework for further empirical analysis. Its major contribution is in the development of a methodology to estimate farmers' potential response to carbon credit incentives in a domestic emissions trading system.

Any policy views, whether explicitly stated, inferred or interpreted from the contents of this report do not necessarily reflect the views or policies of AAFC.

Executive Summary

Soil carbon sequestration is a strategy for dealing with greenhouse gas emissions. This report provides models and analyses of the decision-making process of a farmer who has the opportunity to sign a carbon sequestration contract. In signing the contract, the farmer agrees to adopt a technology or farm management practice that will eventually result in a higher level of soil carbon. Common examples of carbon-conserving technologies include continuous cropping, zero tillage, inclusion of a legume in the crop rotation and converting cropland to permanent grassland.

The farmer's decision is complex because carbon sequestration is inherently dynamic, there are several sources of uncertainty and the decision to sequester carbon is costly to reverse. In addition, the decision to sequester carbon is a once-in-a-lifetime opportunity because once the soil carbon reaches a new equilibrium level, the benefits from the soil contract disappear.

To simplify the analysis, three distinct types of models are used to analyze the problem:

- In the first model, all uncertainty is assumed away and the focus is on the type of technology the farmer should choose. This model draws heavily on the general principles of renewable resource management.

- In the second model, three sources of uncertainty are introduced: the market price of sequestered carbon, the rate of soil carbon accumulation and the opportunity cost of operating with a carbon-sequestration technology rather than the status quo technology. The focus here is on when to adopt a particular technology rather than what type of technology to adopt. This model draws heavily on the general principles of option pricing from the finance literature on investment under uncertainty.

- The third model consists of a conceptual framework rather than a set of mathematical relationships. Key to this framework is the link between carbon sequestration and variability in farm profits and the extent that a change in farm profit variability affects the adoption decision because of risk aversion and prudence.

The first model is a theoretical analysis. The farmer has a range of technologies to consider. At one end, the technology conserves comparatively high levels of carbon and will eventually result in maximum sustainable annual profits. At the other end, the technology conserves comparatively low levels of carbon and results in maximum short-run profits. In the absence of a carbon contract, the size of the farmer's discount rate determines where the optimal technology lies (a lower discount rate shifts the technology toward carbon conservation). A carbon contract offsets the incentive to choose the short-run carbon depleting technology. If the price of sequestered carbon is sufficiently high, then it is possible that the optimal technology conserves more carbon than that which maximizes sustainable annual profits. Given the simple structure of the model, it is always optimal for the farmer to sequester carbon as quickly as possible from the day the contract is signed until the long-run equilibrium is reached.

In the second model of option pricing, the combination of uncertainty and a contracting decision that is assumed irreversible implies that the option to defer the signing decision has value. Because of this option value, the expected net present value (NPV) of the carbon sequestration scheme must be sufficiently positive (i.e. at least as large as the value of the option) before it is optimal for the farmer to sign the contract. For relatively high parameter values of the market price of carbon and for relatively low parameter values for the level of foregone profits from adopting the carbon sequestration technology, immediate investment is optimal. For more intermediate parameter values, the NPV is positive but not sufficiently positive to warrant immediate investment. In this case, the value of the option to delay is about 15 percent.

In the third model, the decision to sequester carbon results in higher risk for three reasons:

- Activities such as summerfallow and conventional tillage tend to reduce risk; eliminating these activities will generally raise risk.

- Investment in any new technology generally involves a period of learning and thus a period of higher risk.

- Investment in a particular carbon sequestration technology such as a zero till drill will generally increase financial leverage and thus financial risk.

If the decision to sequester carbon raises risk, then the more risk averse and prudent the farmer, the higher the needed expected rate of return from the carbon sequestration scheme before the farmer will sign the contract. This risk premium must be added to the option value to get a true sense of the likelihood that a farmer will participate in a carbon sequestration scheme.

There are many strong assumptions that underlie the analysis. These assumptions should be relaxed in future analysis to assess the overall robustness of the results. Similarly, there are many features of the decision to sequester carbon that the current models do not capture. More general forms of analysis are needed to obtain a more robust set of results. The Summary and Limitations section contains a detailed discussion of possible extensions.

Section 1: Introduction

1.1 Background

As countries search for strategies to reduce greenhouse gas emissions, the agriculture sector is receiving considerable attention because of its contribution to atmospheric carbon dioxide and other greenhouse gases. The agricultural carbon cycle is quite simple. Crops convert atmospheric carbon dioxide into carbon compounds (e.g. sugars and carbohydrates). After being used by plants, animals or human beings, or incorporated into the soil as a plant residual and then decomposed by soil microbial activity, the carbon compounds are eventually re-converted into atmospheric carbon dioxide. Because crops are generally intensely managed, changes in management can affect the extent that a particular piece of land contributes carbon dioxide emissions.

The focus of this report is on soil carbon. To get a sense of the relative importance of soil carbon, consider the following carbon cycle for a typical cornfield (Agriculture and Agri-Food Canada 1999, p. 13). In a typical year, the corn plants utilize atmospheric carbon dioxide and produce about 10 tonnes of carbon compounds per hectare of land:

- 3.0 tonnes are returned directly to the atmosphere after being used by the plants
- 2.5 tonnes leave the corn field as the harvested crop
- 4.5 tonnes are incorporated into the soil.

With a stable, long-term corn rotation, the level of carbon in the soil (in the form of organic matter) remains almost constant, implying that each year about 4.5 tonnes of soil carbon are converted into atmospheric carbon dioxide as a result of soil microbial activity.

The level of soil carbon in cropland is currently well below maximum levels (i.e. the carrying capacity) for two reasons:

- Low volumes of residual plant material are returned to the soil each year due to harvest/ grazing and low production volumes.
- The conversion of soil organic matter into atmospheric carbon dioxide is relatively high due to tillage.

Consequently, cropland can potentially serve as a sizeable carbon sink through the rebuilding of soil carbon stocks. Industrial firms, which emit large amounts of carbon dioxide and which are required to reduce emissions, will in many cases be willing to pay farmers to sequester carbon (i.e. remove it from the atmosphere) because it is cost efficient to do so. Agrologists estimate that through the application of best management practices and some acreage conversion, carbon sequestration by U.S. farmers alone could reduce the projected long-term increase in global carbon about seven percent (Sandor and Skees 1999).[1]

The purpose of this report is to examine the economic decision-making process of a farmer who is considering adopting a new technology and/or changing a management regime as a result of an explicit market value placed on sequestered carbon.[2] The details of how the market for carbon emissions will include agriculture have yet to be worked out but farmers may eventually be able to sign carbon sequestration contracts.[3] A sequestration contract would pay a farmer to invest in a specific technology or to adopt a specific management practice (i.e. a design standard) or would pay a farmer according to the volume of carbon that was actually sequestered (i.e. a performance standard). The approach taken in this report is to assume that individual farmers are paid for increases in their stock of sequestered carbon and that the carbon sequestration decision is fully irreversible. The assumption that sequestered carbon can be accurately measured and administered on an individual farmer basis is probably unrealistic but nevertheless is useful as a starting point for this type of analysis. Similarly, the irreversibility assumption is quite strong because presumably the farmer could also use the carbon market by purchasing the right to release carbon back into the atmosphere if it were advantageous to do so.

There are a variety of technologies and management regimes that will increase the level of soil carbon in the long run. (See Agriculture and Agri-Food Canada 1999 for a full discussion.) Minor improvements can be achieved through higher rates of fertilization because more fertilizer implies more plant mass and thus higher volumes of plant residual material being incorporated into the soil. Fertilizing with manure provides additional benefits because the manure itself is organic.

Certainly the types of crops that are grown will also affect the long-run levels of soil carbon. Suppose soil carbon is at a relatively low level and is not changing over time because the land has been in a long-term wheat-fallow rotation. Switching to a continuous crop rotation will generally result in significant increases in soil carbon, especially if one of the crops in the rotation is a legume. In the extreme case, where the cropland is permanently planted to grass, the increase in soil carbon can be very sizeable. Eliminating summerfallow from the rotation can also lead to significant increases in soil carbon because, in the absence of summerfallow, more plant residual is incorporated into the soil. In general, reducing tillage will increase the level of soil carbon because soil organic matter is decomposed and released as carbon dioxide at a relatively slower rate.[4]

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1 Feng, Zhao and Kling (2000) model the benefits of carbon sequestration relative to emissions reduction. They conclude that the value of sequestration is only a fraction of the value of a reduction in emissions unless the sequestration is permanent.

2 There is a growing literature on the economics of forest-based carbon sequestration but very little has been written on soil carbon sequestration for cropland.

3 See Woodward (2000) for a discussion about the various constraints associated with setting up a market for carbon emissions.

4 A management decision that increases the level of soil carbon may indirectly cause other greenhouse gases (e.g. nitrous oxide) to increase. Increased use of manufactured fertilizers is a good example. These external impacts are not considered in this analysis.

1.2 Modeling the Decision to Sequester Carbon

The basic premise is that a farmer will incur a set of additional costs and risks if the farmer wishes to sequester carbon and to sell carbon credits in an emissions market. The extent that the risk-adjusted aggregate benefits of carbon sequestration exceeds the risk-adjusted aggregate cost of carbon sequestration determines the particular carbon-accumulation strategy (if any) that the farmer will choose and the profitability of such a strategy. The costs incurred by a farmer who chooses to accumulate carbon are of three types:

- an up-front cost associated with investing in the new technology or management regime (e.g. zero-till drills)
- additional annual operating costs as a result of the switch in technology or management regime (e.g. the cost of continuous cropping is generally higher than the cost of a wheat-fallow rotation)
- an opportunity cost because the newly adopted crop is of lower value than the status quo crop (e.g. a legume versus wheat).

The net benefit of a particular carbon-accumulation scheme depends on the profitability of the carbon-accumulating crop relative to the conventional crop, the rate of soil carbon accumulation, and the market price of the accumulated carbon.

Three factors characterize the carbon-accumulation management decision and distinguish this management decision from standard farm management decisions:

- The decision is inherently dynamic because decisions made in one period will affect the rate of carbon accumulation (and thus revenues from the carbon market) in future periods.
- The decision to accumulate carbon is largely irreversible. A farmer who signs a carbon sequestration contract cannot release stored carbon back into the atmosphere unless this privilege is purchased in the carbon market. This irreversibility limits future options and may affect the market value of the land.
- The decision has uncertainty. The rate of carbon accumulation and the market price of accumulated carbon are both uncertain.

Also, the profit differential from operating with the carbon-accumulating technology versus the status quo technology is likely to vary considerably over time. The combination of irreversibility and uncertainty implies that the dynamic management decision of the farmer will be complex.

Is there an existing theory that can be used to examine this management decision and to guide the development of an appropriate model for analysis? It seems that no single theory/ model is adequate and that a model which incorporates the three decisions will be highly complex. A good starting point for this analysis stems from the following three classes of models.

The first class of models is designed for optimal management of a renewable resource such as a forest or a fish stock. In these models, the resource stock grows over time at a rate that depends on the management regime and, in the absence of harvest, the carrying capacity of the biological system is eventually reached. However, there are important differences between carbon accumulation and a renewable resource. With the standard renewable resource, the harvest and thus the revenues from the harvest can continue indefinitely. With carbon sequestration, the revenues will accrue to the farmer only while the resource stock is being built up (i.e. payment is made only for net increases in the stock of soil carbon rather than for flows of carbon into the soil). Moreover, because the decision to sequester carbon is largely irreversible, the revenues from the carbon market are in many respects a "once-in-a-life-time" opportunity.

A second class of models that can be used to examine the carbon accumulation involves the theory of investment under uncertainty and irreversibility. The main result that emerges from these models is that the expected net present value (NPV) rule for investment is generally not optimal when the risky decision is irreversible. Delaying the investment decision and collecting more information about the up-and-coming state of the world generally results in a positive option value and thus immediate investment is optimal only if the expected NPV of the investment exceeds this positive option value (rather than zero). A more risky environment generally implies a higher option value and thus for the case of investment in carbon sequestration technologies, the NPV rule alone may be highly misleading.

A third class of models is based on the theory of decision making by prudent and risk-averse decision makers. The decision to adopt a new technology depends critically on three variables:

- the farmer's degree of risk aversion
- the extent that the variability in farm profits and land values changes as a result of adopting the new technology
- the change in farm leverage and financial risk as a result of the new investment.

A higher degree of prudence implies that the farmer will save more (as a contingency for future uncertainty) from a given stream of income. If the farmer faces a probability of bankruptcy, then extreme strategies that are often not consistent with long-run profit maximization may emerge.

1.3 Outline of the Report

In this report, the problem of soil sequestration is approached from three alternative perspectives that correspond to the three models discussed above. The report has six sections. Section 2 has a simple model which assumes away all uncertainty and which draws heavily from Clark's analysis of optimal management of a renewable resource (Clark 1976). In Section 3, three sources of uncertainty are introduced into the model: an uncertain rate of carbon accumulation, an uncertain future market price of carbon, and an uncertain profitability of the carbon-accumulation technology relative to the status quo technology. The stochastic dynamic programming model is developed by assuming that the farmer has only one choice of alternative technology. Thus the decision reduces to choosing when to invest rather than choosing which technology to adopt. In Section 4, simulation results from the stochastic dynamic programming model are presented. The sensitivity of the size of the option to the various parameters of the model provides considerable insight into the farmer's decision-making process. Section 5 has a discussion of the prudence and risk aversion aspects of the decision. A formal analysis of risk aversion and prudence is beyond the scope of this study. The discussion focuses on how and why farm-level risk will change if the farmer chooses to sign a carbon sequestration contract and adopt a new management regime. In Section 6, a summary of the results is provided, the general conclusions are stated and the shortcomings of the analysis are discussed.

Section 2: A Simple Model With no Uncertainty

2.1 Description of the Model

This first model assumes that a set of alternative technologies are available to the farmer and each technology gives rise to a different rate of carbon accumulation and to a different stream of net farm receipts. Because there is no uncertainty, there is no option value associated with delaying the investment decision. Hence, the optimal strategy for the farmer is to choose the technology that maximizes the NPV of the farm. The challenge facing the farmer is to determine how to trade-off the short-run gain in revenues from the carbon market with the long-run cost of operating with a technology that is not optimal in the absence of the carbon contract. The sensitivity of the results to parameters such as the discount rate is also examined.

A farmer is assumed to own a homogenous unit of land with current (date 0) stock of soil carbon given by C(0) = Co . Soil carbon is assumed to change continuously over time until a long-run steady state equilibrium is reached. The instantaneous percentage change in the level of carbon is assumed a linear function of the stock of soil carbon:

$$\dot{C}(t) / C(t) = a - bC(t)$$

where *a* and *b* are positive parameters whose values depend on the general characteristics of the soil and on the technology employed by the farmer (e.g. a wheat-fallow rotation with conventional tillage).[5] To simplify the analysis, assume that a change in technology or farming method affects the intercept *(a)* but does not affect the slope *(b)* .

In particular, assume that $a = \alpha - \beta(Z^M - Z)$ and $b = \alpha / M$ such that

(1) $$\dot{C}(t) = \alpha\left(1 - \frac{C(t)}{M}\right)C(t) - \beta\left(Z^M - Z\right)C(t)$$

[5] Throughout the report, a "dot" over a variable indicates the derivative of that variable with respect to time.

Within equation (1), Z represents a continuous technology choice variable that ranges between 0 and $Z^M > 0$, where Z = 0 is the least carbon preserving technology available to the farmer and $Z = Z^M$ is the most carbon preserving technology available to the farmer.[6] The parameter *M* represents the soil's carbon carrying capacity when $Z = Z^M$ and α and β are rate of change parameters that take on positive values. The first part of equation (1) is the commonly-used logistics function.

The solution to equation (2) is determined by separating the variables and integrating the resulting equation (see Clark 1976, p. 11 for details):

(2) $$C(t) = \frac{\left(\frac{\alpha-\Delta}{\alpha}\right)M}{1+He^{-\alpha t}} \quad where\ H = \frac{\left(\frac{\alpha-\Delta}{\alpha}\right)M - C_0}{C_0} \ and\ \Delta = \beta\left(Z^M - Z\right)$$

Note from equation (2) that regardless of the value of C_0 , the level of carbon always converges toward a steady state (i.e. long-run) value equal to $M(\alpha-\Delta)/\alpha$. A lower value for Z generally results in a lower steady state level of carbon and thus a lower level of total accumulation of carbon from date 0. As well, a lower value of *Z* results in a lower value of *H,* which in turn implies a slower rate of carbon accumulation prior to the carbon stock reaching its steady state level.

To determine the level of Z that will be chosen by the farmer, it is necessary to identify how farm profits depend on *C* and Z. To keep the analysis simple, suppose farm profits (excluding contract payments for soil carbon) can be expressed as:

(3) $$\pi(t) = K + \gamma\left(Z^M - Z(t)\right)C(t)$$

where γ is a non-negative parameter and the parameter *K* can take on either a positive or negative value. This specification of the profit function implies that in the short run (i.e. with soil carbon held fixed), farm profits are maximized by choosing Z = 0 (i.e. by using the least soil-conserving technology available). As well, the higher the carbon- conserving capability of the technology, the lower the level of short-run farm profits. For example, within a particular five-year period, a wheat-fallow rotation is assumed more profitable than continuous cropping, and continuous cropping with conventional tillage is assumed more profitable than continuous cropping with zero tillage. In the long run, when adjustments to *C(t)* are fully accounted for, an intermediate value of *Z* will generally maximize profits, as shown below.

Suppose the farmer enters a long-term contract to accumulate soil carbon. Over a marginal interval of time, revenue from the carbon contract equals $P(t)\dot{C}(t)$ for $\dot{C}(t) > 0$ where *P(t)* is the price of a unit of accumulated soil carbon at time *t*. It is assumed that some form of penalty is in place for net de-accumulation of carbon and that this penalty is sufficiently large such that net de-accumulation is never optimal. *P(t)* may be either constant, increasing or decreasing over time. Regardless of how *P(t)* changes over time, it is natural to assume that *P(t)* eventually converges to a constant long-run value. The following simple function has the desired properties:

6. The choice of technology is not a single dimension choice variable in reality but few additional insights into the problem are generated by considering alternative specifications such as a discrete choice variable.

(4) $$P(t) = P^* + \left(P^0 - P^*\right)e^{-\theta t}$$

Note that P(0) is equal to P^0 and as time increases, *P(t)* approaches *P**, provided that the rate of change parameter θ takes on a positive value. If $P^0 < P^*$, then price increases over time and if the inequality is reversed, then price decreases over time. With $P^0 = P^*$, price is constant over time.

The above specification implies that the farmer is paid only for increases in the stock of soil carbon. An alternative specification is that the farmer is paid a yearly rent on the stock of carbon that has accumulated since the contract was signed. The results of the analysis are quite different for the two alternative cases. Only the first case, where the accumulated carbon is "purchased" from the farmer, is considered in this analysis.

It is now possible to construct an expression for the NPV of profits from the farming enterprise plus revenues from the carbon contract, beginning with time 0 and including all future points in time. If V denotes the present value of this profit and revenue stream and b denotes the discount rate, then the appropriate expression is:

(5) $$V = \int_0^\infty e^{-\delta t}\left(K + \gamma\left(Z^M - Z\right)C(t) + P(t)\dot{C}(t)\right)dt$$

When choosing technology Z to maximize V in equation (5), the farmer is constrained by the equation of motion for soil carbon that is given by equation (1).

It is important to ask whether it is reasonable to assume that the farmer has an infinitely long time horizon. Suppose the farmer in question plans to operate until date *T* and then retire. At date *T,* in a competitive land market, the bid price for the farmer's land should equal the value implied by equation (5) with *C(*0) and *P(0)* for the new farmer equal to *C(T)* and *P(T),* respectively, from the original farmer. If the original farmer's objective is to maximize the discounted stream of profits from farming plus the discounted selling price of the land, then the objective function should be rewritten as

(6) $$V = \int_0^T e^{-\delta t}\left(K + \gamma\left(Z^M - Z\right)C(t) + P(t)\dot{C}(t)\right)dt + e^{-\delta T}\int_T^\infty e^{-\delta(\tau - T)}\left(K + \gamma\left(Z^M - Z\right)C(t) + P(\tau)\dot{C}(\tau)\right)d\tau$$

Note that equation (6) reduces to equation (5). Thus, regardless of the time horizon of the original farmer, the problem is solved with an infinite time horizon in mind.

It is convenient to solve the problem implied by equation (5) using a calculus of variations approach (see Clark 1976 for full details). Begin by defining

$F(C) \equiv \alpha\,(1 - C/M)C$

and using equation (1) to substitute $[F(C) - \beta Z^M C - \dot{C}\,]/\beta$ for $-CZ$ in equation (5):

(7) $$V = \int_0^\infty e^{-\delta t}\left(K + \frac{\gamma}{\beta}F(C(t)) - \left(\frac{\gamma}{\beta} - P(t)\right)\dot{C}(t)\right)dt$$

The Euler equation that defines a singular solution to this problem is given by

(8) $$\frac{\partial\left[K + \frac{\gamma}{\beta}F(C)\right]e^{-\delta t}}{\partial C} = \frac{\partial\left[-\left(\frac{\gamma}{\beta} - P(t)\right)e^{-\delta t}\right]}{\partial t}$$

If the appropriate expressions are substituted for *F(C)* and *P(t),* then the two derivatives in equation (8) can be fully evaluated. After solving the resulting expression for the optimal trajectory of soil carbon C^* *(t) ,* the following expression emerges:

(9) $$C^*(t) = \frac{\gamma\alpha - \delta(\gamma - \beta P^*) - (\delta + \theta)\beta(P^* - P^0)e^{-\theta t}}{2\gamma\alpha}M$$

Equation (9) is referred to by Clark (1976) as the singular solution to the problem. It is singular in the sense that $C^*(0)$ is typically not equal to the actual starting value for *C(t),* which is a parameter and which is denoted C_0. Clark establishes that with a linear variation model (the current model is considered a linear variation model), the optimal solution is to proceed from the starting value of the steady state variable (in this case C_0) to the path of the state variable implied by the singular solution (in this case *C*(t))* as rapidly as possible, subject to the constraints on the control variable (in this case $\dot{C}$ *(t)).* This process is discussed in greater detail below.

To complete the solution to the problem, it is necessary to derive the specific technology that will ensure that soil carbon follows the path implied by *C*(t).* Begin by taking the time derivative of equation (9) to obtain an expression for $\dot{C}^*(t)$:

(10) $$\dot{C}^*(t) = \frac{\beta M\theta}{2\gamma\alpha}(\delta + \theta)(P^* - P^0)e^{-\theta t}$$

Equations (9) and (10) can now be substituted into equation (1) and the resulting expression solved for Z^* *(t)* to obtain

(11) $$Z^*(t) = Z^M - \frac{0.5}{\beta}\left[\alpha - \frac{\delta}{\gamma}(\gamma - P^*) - \frac{(\delta + \theta)}{\gamma}(P^* - P^0)e^{-\theta t}\right] + \Phi(t)$$

where

(12) $$\Phi(t) = \frac{\dot{C}^*(t)}{C^*(t)} = \frac{\theta(\delta + \theta)(P^* - P^0)e^{-\theta t}}{\gamma\alpha - \delta(\gamma - \beta P^*) - (\delta + \theta)(P^* - P^0)e^{-\theta t}}$$

Equations (11) and (12) describe the technology the farmer should employ as a function of time.

2.2 Interpreting the Long-run Steady State Results

In the long run, the optimal level of soil carbon converges toward a steady state value C^{**} that is independent of the initial level of soil carbon. This result can be established by letting time increase indefinitely in equation (9) and then noting that $C^*(t)$ converges toward

$$(13) \qquad C^{**} = \frac{\gamma\alpha - \delta(\gamma - \beta P^*)}{2\gamma\alpha} M$$

To understand equation (13) it is useful to discuss first the notion of maximum annual sustainable profit.

2.3 Maximum Annual Sustainable Profit

The concept of a sustainable economic yield is important in most problems involving renewable resources. In this report, it is of interest to determine the maximum level of annual profit that could potentially be sustained in the long run. The annual level of profit associated with the optimal solution is generally less than the maximum annual sustainable profit because of the effects of discounting. As well, maximum annual sustainable profit does not depend on the parameters of the carbon contract because the revenues from the carbon contract do not accrue in the long run (i.e. once soil carbon reaches a new steady state, payments from the carbon contract are terminated).

In the steady state, $\dot{C}(t) = 0$ and thus using equation (1), $\beta(Z^M = Z)C = \alpha(1 - C/M)C$. This equation can be rearranged and written as $C = M - (\beta/\alpha\ (Z^M - Z)M$. If both sides of this equation are multiplied by $\gamma\ (Z^M$ - Z)and increased by the additive term K, the following expression results:

$$(14) \qquad K + \gamma(Z^M - Z)C = K + \gamma(Z^M - Z)\left[1 - \frac{\beta}{\alpha}(Z^M - Z)\right]M$$

The left side of equation (14) is annual farm profits (excluding the carbon contract), expressed as a function of both Z and C . The right side of equation (14) can therefore be interpreted as an expression for annual sustainable farm profits, expressed as a function of Z only.

Figure 1 shows a graph of equation (14). Note that for relatively small values of Z (i.e. strong carbon depleting technologies) or for relatively large values of Z (i.e. strong carbon conserving technologies), the level of annual sustainable farm profits is relatively small. In the first case, profits are small because soil carbon will erode to a comparatively small base, thus making the land relatively unproductive. In the second case, profits are small because of the relatively high cost (or low

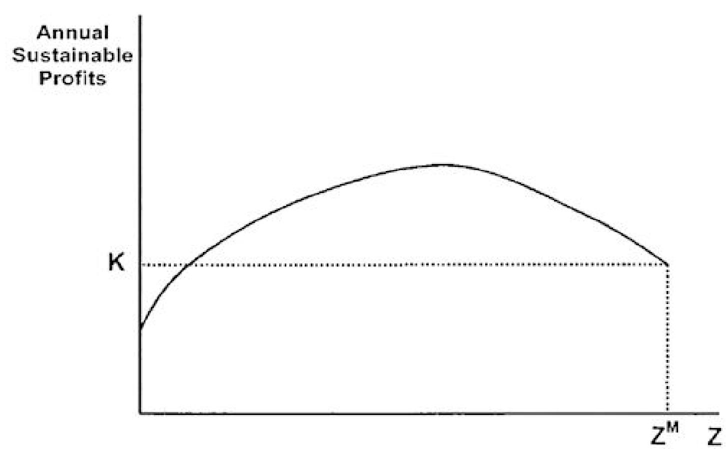

Figure 1: Annual Sustainable Profits as a Function of Technology.

productivity) of operating with a non-intensive technology. Maximum sustainable profits occur for intermediate technologies.

To obtain an expression for the technology that maximizes annual sustainable farm profits, it is sufficient to maximize the right side of equation (14) with respect to Z. It is easy to show that $Z^S = Z^M - 0.5\,\alpha/\beta$ is the appropriate expression. It is perhaps more useful to examine the steady state level of soil carbon when $Z = Z^S$. Recalling that $\beta(Z^M - Z) = \alpha\,(1 - C/M)$ in steady state, it is easy to show that the steady state level of soil carbon that is associated with maximum annual sustainable farm profits can be expressed as:

(15) $C^S = M/2$

In other words, with the simple functional forms assumed for this model, annual sustainable profits are maximized when soil carbon is at 50 percent of the level that would result with the $Z = Z^M$ technology.[7]

2.4 Comparing C^{**} and C^S

Equation (13) provides an expression for C^{**}, which is the optimal level of soil carbon for the farm in the steady state and equation (15) provides an expression for C^S, which is the level of soil carbon that maximizes annual sustainable profits. A comparison of the two expressions shows that $C^{**} = C^S$ when the discount rate δ equals zero and $C^{**} < C^S$ when $\delta > 0$. Moreover, the price of the carbon contract has no impact on C^{**} when $\delta = 0$ and C^{**} is higher with a higher price for the carbon contract when $\delta > 0$. The intuition of these important results is now discussed.

If the farmer (who does not sign a carbon contract) considers only the short run, then profits are maximized when Z = 0. If the farmer wishes to maximize the annual sustainable level of profits, then $Z^S = Z^M - 0.5\alpha/\beta$ should be chosen. With $\delta = 0$, maximizing the NPV of the carbon contract is equivalent to maximizing annual sustainable profits, and thus the optimal technology in this case is $Z^{**} = Z^S = Z^M - 0.5\alpha/\beta$. With $\delta > 0$, the farmer places relatively higher weight on short-run profits and thus the optimal technology shifts somewhere between $Z^{**} = 0$ and $Z^{**} = Z^S$. This outcome explains the result that the optimal steady state level of soil carbon C^{**} is less than the level of soil carbon that maximizes annual sustainable profits C^S when $\delta > 0$.

The carbon contract provides the revenues only until a new steady state level of soil carbon is reached. Consequently, the carbon contract affects the farmer's short-run but not long-run soil management incentives. With $\delta = 0$, the farmer completely ignores short-run incentives and will thus choose $Z = Z^S$ and not sign the contract, regardless of the contract price of carbon. With $\delta > 0$, the farmer responds to the short-run incentives and thus now accounts for the contract price of carbon when choosing Z. Equation (13) shows that with $\delta > 0$, the carbon contract will offset the farmer's incentive to choose Z = 0 in the short run. A higher value of P^* diminishes the short-run incentives and therefore shifts the optimal level of soil carbon upward toward C^S. Equation (13) shows that for

7. This 50 percent result is specific to this simple model and should not be viewed as an estimate of the optimal level of soil carbon in the real world.

a sufficiently large value of P^*, the short-run incentive to set Z = 0 may be more than offset by the carbon contract such that the optimal steady state level of soil carbon exceeds C^S. In other words, the carbon contract may be sufficiently attractive in the short run such that the farmer is willing to operate in the long run with a technology that is relatively inefficient (i.e. would not be chosen in the absence of a carbon contract). These specific results are consistent with Clark's (1976) observation that the general rule for efficient management of a renewable resource is to choose the resource stock such that the marginal immediate gain is equal to the present value of the stream of marginal future loss.

2.5 Interpreting the Short-run Path Results

Now consider the optimal technology and the optimal level of soil carbon for the time period between date 0 and when the long-run equilibrium is reached. Equation (9) shows that if the price of carbon is expected to remain constant over time (i.e. $P^0 = P^*$), then $C^*(t)$ is constant and equal to C^{**}. As explained above, $C^*(t)$ is a singular solution to the problem and it implicitly assumes that the farmer will adjust from the initial stock value of carbon C_0 to the optimal path $C^*(t)$ as rapidly as possible. In a more general model that incorporates various non-linearities and adjustment costs, the path from C^0 to $C^*(t)$ would generally not be the most rapid approach. An interesting implication of the most rapid approach result is that it implies that farmers who sign carbon contracts should adopt a technology that accumulates soil carbon as fast as possible and once the long-run equilibrium is reached, they should adopt the long-run steady state technology and no more revenues from the carbon contract will be forthcoming.

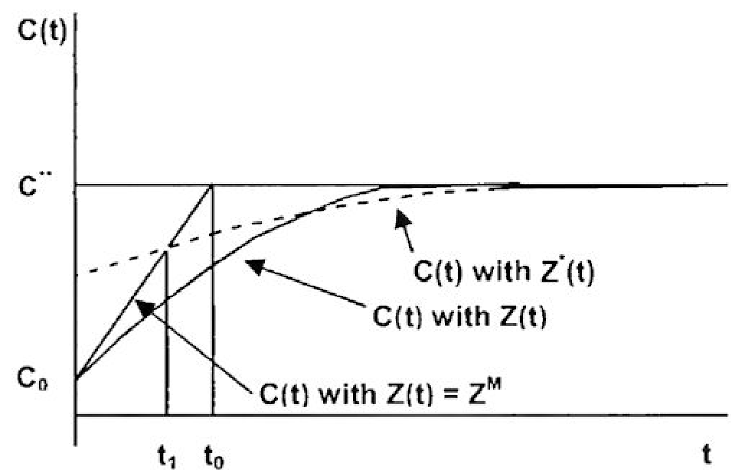

Figure 2: Optimal Path of Soil Carbon With Constant or Rising Carbon Price.

Figure 2 shows the case where the singular solution to the problem is $\{Z^*(t) = Z^{**}, C^*(t) = C^{**}\}$ and the farmer chooses Z^M so as to move from C_0 units of carbon to C^{**} units of carbon as quickly as possible. At time t_0, the farmer switches from technology Z^M to technology Z^{**}. Suppose switching technology at time t_0 is costly. The farmer may then wish to choose technology Z^{**} from date 0 onward and allow soil carbon to increase more gradually and to converge toward the long-run steady state value C^{**}. This solution results in the carbon contract providing the farmer with relatively smaller annual payments but the payments are forthcoming for a longer period of time. Although it has not been formally worked out, the optimal solution when the farmer is constrained to choose just one technology may result in a long-run level of soil carbon that is slightly to moderately above C^{**}.

Now consider the case where the price of carbon is expected to rise over time (i.e. $P^0 < P^*$). Equation (9) shows that $C^*(t)$ takes the form of a concave function that slowly converges toward the long-run equilibrium value C^{**}. This $C^*(t)$ function is shown as the dashed line in Figure 2. Equation (10) shows that the rate of increase in $C^*(t)$ is proportional to θ. This result makes sense because when the price of carbon is rising quickly (i.e. θ takes on a relatively high value), then the rate of carbon accumulation should also rise quickly such that the largest volumes of accumulated carbon will tend to be sold at the highest price. As before, the optimal solution calls from the most rapid approach to $C^*(t)$ from C^0. Thus, the optimal solution entails using technology Z^M from date 0 to date t_1 and then permanently switching to technology $Z^*(t)$.

The above solution is unrealistic because with $P^0 \neq P^*$, the optimal technology path $Z^*(t)$ is not constant between date 0 and the point of long-run equilibrium. For the case of rising carbon prices, the optimal path calls for a decreasing value of Z over time. For most farms, it is not feasible to change technologies constantly and thus a constrained second-best solution will generally have to be implemented. If switching technologies once or twice is reasonable, then the farmer should switch gradually to technologies that increasingly accumulate carbon. If switching technologies is costly, then, as in the previous case, the farmer may want to choose a technology that is somewhat near to Z**.

Finally, consider the case where $P(t)$ declines over time. This case is not formally analyzed but the solution seems rather intuitive. With declining $P(t)$, the optimal path for $C(t)$ will decline over time until it converges upon C^{**}. The problem is that it will generally take considerable time for the farmer to move from a starting position of soil carbon C_0 to the optimal path $C^*(t)$ because $C^*(t) > C^{**}$. In the most likely case, $C^*(t)$ will be near C^{**} by the time the soil carbon has been built up to the level of C^{**}. In this case, the solution to the problem is the same as when the price was assumed constant over time.

Section 3: A Model of Irreversible Investment Under Uncertainty

3.1 Purpose of the Section

The analysis of Section 2 helped identify some general economic principles of the carbon accumulation problem. However, a major deficiency in the analysis is the absence of uncertainty in three key variables:

- the contract price of soil carbon
- foregone farm profits when substituting one technology for another
- the rate of soil carbon accumulation.

The purpose of this section is to incorporate these three forms of uncertainty into a simple option value model of carbon accumulation. (See Dixit and Pindyck 1994 for a general description of this type of model.) The model assumes that the farmer has a choice of maintaining a conventional technology or of signing a carbon contract and of adopting a single alternative technology that will result in soil carbon accumulation. The decision facing the farmer reduces to choosing when (if ever) to adopt the new technology. The analysis is simplified by assuming that the adoption decision is irreversible. De-accumulation of carbon would require the farmer to purchase carbon credits and it would be quite complex to incorporate this feature into the model.

3.2 Specification of Uncertainty

The decision regarding when to sign the carbon contract depends crucially on the form of the uncertainty. A common specification of uncertainty in an option value model is the Brownian Motion with drift stochastic processes. This Brownian Motion process is utilized to incorporate the three forms of uncertainty, which have been identified above.

Contract price of carbon - The Brownian Motion equation that governs how the contract price of carbon evolves over time can be written as

(16) $$dP = \mu_P P dt + \sigma_P P dS_P$$

Equation (16) indicates that over the next infinitesimal time interval *dt,* the percent change in price (i.e. dP/P) is equal to a deterministic amount μ_P, and a random amount $\sigma_P\, dS$, where σ_P is a constant, $dS_p = s_P\,(dt)^{0.5}$ and s_p is a standard normal random variable. It is common to refer to μ_P as the drift parameter and σ_P as the variation parameter. The larger the value of μ_P, the greater the tendency for the *P(t)* series to drift upward over time. The greater the value of σ_P, the greater the variability in *P(t)* over time. Note that the specification of equation (16) is such that it is not possible for *P(t)* to take on a negative value.

Foregone farm profits - Let $\pi(t)$ denote farming profits (excluding revenues from the carbon contract) at time *t* that are lost as a result of operating with the new technology rather than the conventional technology. It is assumed that the conventional technology is relatively land intensive and generates profits that are always at least as high as the farming profits that can be earned with the alternative technology. In other words, $\pi(t) \geq 0$. It is also assumed that $\pi(t)$ follows the Brownian Motion with drift stochastic process:

$$(17) \qquad d\pi = (\mu_\pi \pi)dt + (c_\pi \pi)dS_\pi$$

In this case, $dS_\pi = s_\pi\, dt^{0.5}$ where s_π is a standard normal random variable that is independent of s_P. The independence of s_P and s_π implies that *P(t)* and $\pi(t)$ are independently distributed random variables.

Rate of carbon accumulation - The third source of uncertainty to be incorporated into the model is the rate of carbon accumulation. The proposed equation is a stochastic version of the logistics function that was specified in Section 2. Specifically,

$$(18) \qquad dC = \mu_C \left(1 - \frac{C}{M}\right) C dt + \sigma_C . C dS_C$$

As before, $dS_C = s_C\, dt^{05}$ where s_C is a standard normal random variable that is independent of both s_P and s_π. It can be seen from equation (18) that the stock of soil carbon will stochastically evolve toward the carrying capacity *M.* In the long run, soil carbon will randomly fluctuate around *M.*

3.3 Probability Transition Matrix

In a simple option model, analytical solutions are possible. However, a large class of models (of which the current model is a member) must be solved using discrete stochastic dynamic programming techniques. The key component of the discrete stochastic dynamic programming model is the probability transition matrix. To build this matrix, it is assumed that each state variable can take on one of *n* values *(n* can be different for the different variables but in the current analysis, *n* is assumed common across all three variables). Let *w* denote the difference between two neighboring values in a particular state variable sequence (*w* will generally take on different values for the different variables). Because each variable has a minimum value of zero, it follows that the sequence of values for a particular variable is {w/2, 3w/2, 5w/2... (2n - 1)w/2 }.[8] Using this procedure, sets of discrete values can be constructed for each of the three state variables within the model.

8. The midpoint of each interval in the sequence is used.

Suppose at time t, $\pi(t)$ takes on the i^{th} value in the farm profit sequence, $P(t)$ takes on the j^{th} value in the carbon price sequence and $C(t)$ takes on the k^{th} value in the carbon stock sequence. Now define $Pr(i, j, k, a, b, c)$ as the probability that the state variable triplet moves from position (i, j, k) to position (a, b, c) (i.e. farm profit moves from the i^{th} value to the a^{th} position in the sequence and so forth). Because the three state variables are independently distributed, it is sufficient to calculate independently the probabilities that the farm profit will move from position i to position a, the carbon price will move from position j to position b, and the soil carbon will move from position k to position c. Then multiply the three independent probabilities together to determine the joint probability that all three moves will happen concurrently.

To calculate the probability that the carbon price moves from position j to position b, a discrete version of equation (16) is used: $P_t = (1 + \mu_P)P_{t-1} + \sigma_P P_{t-1} s_P$. Let $\Phi(P, \mu, \sigma)$ denote the cumulative density function for a normally distributed random variable P with mean μ and standard deviation σ. It then follows that the probability that the price variable with value (2j - 1)w/2 in period t takes on a value of (2b - 1)w/2 in period $t + 1$ is equal to $\Phi(P_1, P_M, P_S) - \Phi(P_0, P_M, P_S)$, where $P_1 = (2b-1)w/2 + w/2$, $P_0 = (2b-1)w/2-w/2$, $P_M=(1 + \mu_p)(2j-1)w/2$, and $P_S = \sigma_P (2j - 1)w/2$. If position b happens to be the last position in the price sequence, then it is necessary to add to the previously-calculated probability the cumulative probability that P_t will exceed (2n - 1)w/2 + w/2 . This cumulative probability is given by $l - \Phi(P_N, P_M, P_S$, where $P_N = (2n - 1)w/2 + w/2$.

Calculate the probability transition values for the farm profit and the carbon stock sequence is similar to the procedures previously described for the carbon price sequence. The probability transition measure $Pr(i, j, k, a, b, c)$ can be placed in a six-dimensional matrix with n cells in each dimension to obtain the full probability transition matrix, denoted Pr. Regardless of the time period, this matrix identifies the probability that the state variable triplet will move from one set of values to another set of values.

3.4 Expected Profits When the Contract is Signed

In addition to constructing a transition matrix to solve the problem, it is necessary to calculate the NPV of the stream of net profits that the farmer expects when the carbon contract is signed and the new carbon-accumulating technology is adopted. Profits are measured in changes rather than levels (i.e. the change in profits relative to the status quo is measured and tracked). As discussed previously, it is appropriate to assume that the farmer maximizes an infinitely long time horizon when making management decisions. However, the stochastic dynamic programming problem is most easily solved with a finite horizon. Thus, the approach taken here is to solve the problem with a finite time horizon of length T and then set T equal to a relatively large value to obtain an approximate solution to the infinite-period problem. Because of the effects of discounting, the approximation error associated with this approach can be made arbitrarily small.

Suppose the state is $\{i, j, k\}$ in period $T-1$ (i.e. π_{T-1} takes on the i^{th} value in the farm profit sequence and so forth) and the farmer chooses to sign the contract and to adopt the technology. The present value of the expected gain in net profits over the next and final period as a result of the adoption decision is equal to $V(i, j, k; T-1) - I$ where I denotes the fixed cost of the investment and[9]

9. To conserve notation, let X^i equal the value of generic variable X when X takes on the i^{th} value in its sequence.

(19) $$V(i,j,k;T-1)=\delta\sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n}\Pr(i,j,k,a,b,c)\left[\left(C^{c}-C^{k}\right)P^{b}+\pi^{a}\right]$$

The discounting parameter b is equal to the inverse of one plus the discount rate. Equation (19) can be used to compute values for *V(i, j, k; T-1)* for all values of *i, j* and *k.* In period *T* - 2 , the present value of the expected gain in net profits over the two remaining periods can be expressed as

(20) $$V(i,j,k;T-2)-I=\delta\sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n}\Pr(i,j,k,a,b,c)\left[\left(C^{c}-C^{k}\right)P^{b}+\pi^{a}+\delta V(a,b,c;T-1)\right]$$

The iterative solution technique that is implied by equations (19) and (20) can be used repeatedly to calculate values for the *V* function for all values of *i, j* and *k* and for all values of *t* between 0 and *T*-1.

3.5 Stochastic Dynamic Programming Solution

The decision facing the farmer is when to sign the carbon contract and when to invest amount 1 so as to begin using the carbon-accumulating technology. With the traditional investment model, the farmer will sign the carbon contract at the point where NPV is positive. For example, if the state variables take on values {i, *j, k*} at date 0 and if V(*i, j, k,* 0) -*I* > 0, then traditional analysis implies that the contract should be signed immediately. With traditional analysis, equations (20) and (21) are sufficient to solve the entire problem. Modern investment theory indicates that when the decision is irreversible, then the investment should occur only if *V(i, j, k, 0) -I* is sufficiently positive. In particular, V(*i, j, k, 0)- I* must exceed the option value associated with delaying the investment and collecting additional information about the state of the world. This type of problem is referred to as "optimal stopping" and it is generally solved using stochastic dynamic programming techniques.

If in period *t,* the farmer delays the decision to sign the carbon contract by one period, the status quo technology is necessarily maintained. In this case, $\pi(t)$ = 0 and revenues from the carbon contract also equal zero. The value of the option to sign the contract at a future date is equal to the maximum of the current NPV of the contract and the discounted expected value of the option to sign the contract in period *t* + 1, assuming that the optimal decision is made in period *t* + 1. Let *F(a, b, c; t + 1)* denote the value of the option to sign the contract in period *t* + 1 assuming that the triplet of state variables take on values corresponding to *a, b* and *c.* In period *t,* with state {i, *j, k*}, the farmer will choose to delay the carbon contract decision if

(21) $$V(i,j,k;t)-I<\delta\sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n}\Pr(i,j,k,a,b,c)F(a,b,c;t+1)$$

From equation (21) and the above discussion, it follows that

(22) $$F(i,j,k;t+1)=Max\left\{V(i,j,k;t+1)-I,\ \delta\sum_{a=1}^{n}\sum_{b=1}^{n}\sum_{c=1}^{n}\Pr(i,j,k,a,b,c)F(a,b,c;t+2)\right\}$$

For the last period, *F(a, b, c; T) = V(a, b, c; T) —1.* This terminal condition, together with the recursive relationship implied by equation (22), can be used to calculate a value for *F(i, j, k; t)* for all values of *i, j* and *k* and for all periods of time between 0 and *T.* Equation (22), which is referred to as the Bellman equation, is of key importance for the stochastic dynamic programming process.

In optimal stopping problems, it is common to divide the state space into "continue" and "stop" regions and to illustrate these two regions on a diagram. First consider the simple case where the only source of uncertainty is the contract price. Intuitively, if the price of carbon increases to a sufficiently high level, then the farmer will stop waiting and sign the contract. In Figure 3, the critical value for the carbon price, denoted P_t*, is graphed as a function of time. At a given point in time, if the actual carbon price (which is random) exceeds P_t*, then the farmer should sign the contract. If $P_t \leq P_t$*, then waiting is optimal. The set of solution values for P_t* is determined by setting $V(P, t) - I = F(P, t)$ and solving for P where $V(P, t)$ and $F(P, t)$ are defined analogous to the $V(i, j, k; t)$ and $F(i, j, k; t)$ functions that were specified above.

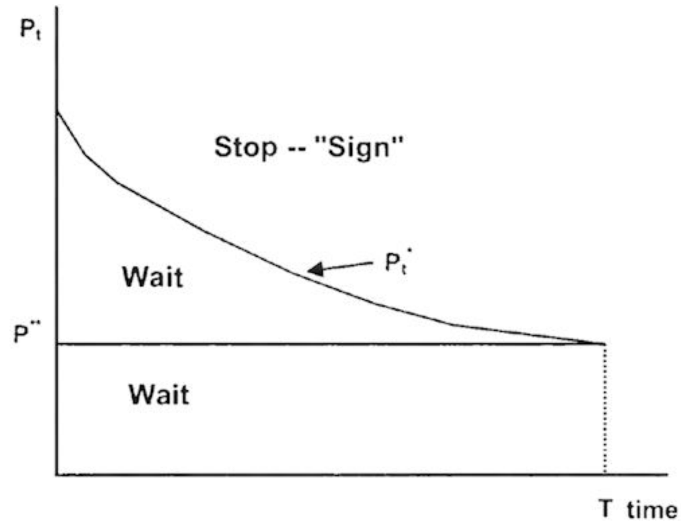

Figure 3: Illustration of Results in Simple Version of Model.

Note in Figure 3 that P_t* declines over time and equals the critical price *P*** at time *T.* Price *P*** is the price for which the expected NPV of signing the contract and adopting the new technology (i.e. V) is just equal to zero. In other words, if the NPV rule were used for the contract decision, then *P*** is the critical price, above which the contract would be signed and below which the contract would not be signed. The difference between P_t* and *P*** reflects the option value for waiting and collecting more information about the state variable. This option value is largest when the remaining time horizon is largest and shrinks to zero as the system approaches time *T.* The actual value of the option is equal to $V(P, t)$ when $V(P, t) = F(P, t)$ and $V(P, t) \geq 0.$

In the current problem, there are three stochastic state variables to consider *(P, π* and *C)* and an infinite rather than finite time horizon. With the status quo technology, soil carbon is not expected to change over time and thus after making the adjustment for discounting and allowing for an infinitely long time horizon, the calculated values for $V(i, j, k; t)$ and $F(i, j, k; t)$ are necessarily independent of the time period. Consequently, the decision rule is also independent of the time period, unlike that illustrated in Figure 3. Because no reference to time is necessary, all results will be presented as of date 0.

To illustrate the "continuation" and "stopping" regions for this more general problem, it is useful to treat the stock of soil carbon as a fixed parameter and to construct graphs with $\pi(0)$ and P(0) on the respective axes. Let the solution to the function $G(\pi, P) = 0$ implicitly specify combinations of values for $\pi(0)$ and $P(0)$ which ensure that $(\pi(0), P(0), C^0;0) = 0$. In other words, if $\pi(0)$ and P(0) satisfy $G(\pi, P) = 0$, then the NPV of the farmer's expected gain in profits as a result of signing the carbon contract is equal to zero. In traditional investment analysis, the $G(\pi, P)$ function would be used as the decision rule. Let the solution to $G^* (\pi, P) = 0$ imply combinations of values for $\pi(0)$ and P(0) which ensure that $(\pi(0), P(0), C^0;0) = F(\pi (0), P(0), C^0; 0$. If $\pi(0)$ and P(0) are such that $G^*(\pi, P) = 0$, then at date 0 the farmer is just indifferent between signing the contract and deferring the decision by one period.

The $G(\pi, P) = 0$ and $G^*(\pi, P) = 0$ schedules are graphed in Figure 4. First consider the $G(\pi, P) = 0$ schedule. It is upward sloping because a higher initial loss from operating with the alternative technology versus the conventional technology, as measured by $\pi(0)$, implies that a higher value of P(0) is needed to ensure that the expected NPV of signing the carbon contract is zero. The area below (above) the $G(\pi, P) = 0$ schedule represents all the combinations for $\pi(0)$ and $P(0)$ for which the expected NPV of the carbon contract is positive (negative). There are a variety of parameters that affect the location of the $G(\pi, P) = 0$ schedule. These sensitivity results are discussed in Section 4.

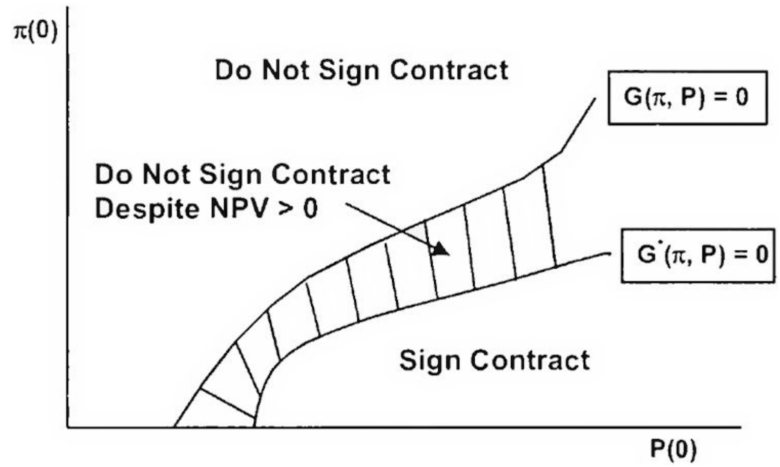

Figure 4: Illustration of Results in General Case.

Now consider the $G^*(\pi, P) = 0$ schedule in Figure 4. It is upward sloping for reasons similar to why the $G(\pi, P) = 0$ schedule is upward sloping. More importantly, the $G^*(\pi, P) = 0$ schedule lies below the $G(\pi, P) = 0$ schedule because of the implied option value. Specifically, the farmer should sign the contract only if the $\pi(0)$ and P(0) combination lies below the $G^*(\pi, P) = 0$ schedule. The region between the $G(\pi, P) = 0$ and $G^*(\pi, P) = 0$ schedules represents $\pi(0)$ and $P(0)$ combinations for which the farmer should not sign the contract, despite the existence of a positive expected NPV from doing so. The primary purpose of the empirical analysis is to get a sense of the likely magnitude of this implied option value and to examine the sensitivity of the option value estimates to changes in the various parameters of the model. If option value estimates are in the range of 10 to 20 percent, then it is fair to conclude that the NPV rule is relatively severely biased.

Section 4: Simulation Results

4.1 Data and Procedures for Estimating Model Parameters

The purpose of the simulation is to illustrate the general features of the solution to the stochastic dynamic programming problem. Rather then choosing parameter values that are intended to reflect a particular situation (i.e. the adoption of a particular technology for a particular soil type), parameter values that are thought to be "reasonable" for a typical situation are used. For a typical piece of cultivated land that currently has 40 to 60 tonnes of organic carbon per hectare, the gain in soil carbon from the adoption of a zero till system might range from zero to 10 tonnes/hectare after a ten-year period (Agriculture and Agri-Food Canada 1999, p. 15-17). Based on these data, the initial level of carbon in the simulation model was set equal to 50 tonnes/hectare, the carrying capacity M was set equal to 60 tonnes/hectare and the maximum possible value of soil carbon (for the purpose of constructing the transition matrix) was set equal to 62 tonnes/hectare. A value of 0.10 was chosen for the soil carbon drift parameter μ_C and a value of 0.04 was chosen for the soil carbon variability parameter σ_C. With these parameter values, soil carbon levels typically increase and fluctuate around the soil's carrying capacity after 20 to 25 years of continuous application of the new technology. As well, the average level of soil carbon after 15 years is about 57 tonnes/hectare with a standard deviation of 3.2 tonnes/hectare.

The market price for soil carbon is highly uncertain at this point in time. Sandor and Skees (1999, p. 14) report that previous estimates of carbon emissions range from US$15/ton to US$348/ton. They go on to suggest that a more conservative range of US$20/ton to US$30/ ton was used in recent modeling exercises. Based on these latter estimates, the feasible range of carbon price was set equal to Cdn$20/tonne to Cdn$50/tonne. Initially, the drift parameter for carbon price μ_P was set equal to zero and the variability parameter σ_P, was set equal to 0.3. With these values and beginning with a carbon price equal to $35/tonne at year 0, the average price of carbon by year 15 is equal to $32/tonne and the standard deviation around this average is $11/tonne.

There is also considerable uncertainty regarding the reduction in farm level profits as a result of signing the carbon contract and adopting the alternative technology. Recall that there are two components to this cost: an up-front initial investment I and a yearly operating cost $\pi(t)$. Rather than attempting to specify investment costs on a per hectare basis, it is assumed that the farmer rents the new technology on a year-to-year basis. Assume that the cost of these services relative to the status quo ranges from zero to $30/hectare. These values are somewhat arbitrary but they do yield a "reasonable" set of results. For the base case simulations, the foregone profit drift parameter μ_π is set equal to zero and the foregone profit variation parameter σ_P is set equal to 0.3. With these values and beginning with a level of foregone profits equal to $15/hectare, the average level of foregone profits by year 15 is $10.6/hectare and the standard deviation is $9.1/hectare.

The remaining parameters to be specified are the number of discrete intervals into which each state variable is divided *(n),* the total number to years in the simulation *(T)* and the discount factor (δ). Computing time grows exponentially with *n* and *T* so relatively small values for these variables must be chosen. For the simulation results in this report, *n* = 15 and *T* = 15. Sensitivity analysis reveals that the length of the time horizon has very little impact on the date 0 results once *T* reaches 10. The discount rate was set equal to 0.95.

4.2 Base Case Results

The results of the simulation model are now presented, beginning with the base case scenario. Table 1 reports the NPV of the change in expected farm profits as of date 0 from signing the carbon contract (ignore the shading for the moment). The NPV is reported for varying values of date 0 carbon prices (column headings) and for varying values of the level of date 0 foregone profits (row headings). In other words, the cell entry of $81/hectare, which corresponds to the fifth column and the fifth row, assumes that at the time the farmer makes the decision about the carbon contract, the price of carbon is $28.60/ tonne and the yearly net cost of renting the alternative technology is $8.60/hectare. The NPV of $81/hectare accounts for how these two variables are likely to evolve in the future and how the current stock of soil carbon (always assumed to be at 50 tonnes/hectare at date 0) is likely to evolve in the future according to equation (18). The NPV results reported in Table 1 equal the discounted stream of expected *change* in profits from signing the contract at date 0 (not the expected *level* of profits). Ignoring option value considerations (discussed next), a positive value in Table 1 reflects the extent that the market value of the land is expected to increase if a carbon trading scheme were implemented.[10] As expected, the larger the value of P(0) and the lower the value of π(0), the higher are the NPV figures.

Foregone Profits at Date 0 ($/hect)	Price of Carbon at Date 0 ($/tonne)														
	20.0	22.1	24.3	26.4	28.6	30.7	32.9	35.0	37.1	39.3	41.4	43.6	45.7	47.9	50.0
0.0	144	149	154	160	166	172	177	183	188	192	196	200	203	205	207
2.2	122	127	132	138	144	150	156	161	166	170	174	178	181	183	186
4.3	101	105	111	116	122	128	134	139	144	149	153	156	159	162	164
6.4	80	84	90	95	101	107	113	118	123	128	132	135	138	141	143
8.6	60	64	70	75	81	87	93	98	103	108	112	115	118	121	123
10.7	**41**	45	51	57	62	68	74	79	84	89	93	96	99	102	104
12.9	**23**	**28**	**33**	**39**	45	51	57	62	67	71	75	79	82	84	87
15.0	**7**	**12**	**17**	**23**	**29**	**35**	41	46	51	55	59	63	66	68	71
17.1	-7	-2	**3**	**9**	**15**	**20**	**26**	**31**	**36**	41	45	48	51	54	56
19.3	-20	-15	-10	-4	**2**	**8**	**13**	**19**	**23**	**28**	**32**	35	38	41	43
21.4	-31	-27	-21	-15	-10	-4	**2**	**7**	**12**	**17**	**21**	**24**	**27**	**30**	**32**
23.6	-41	-36	-31	-25	-19	-13	-8	-2	**3**	**7**	**11**	**15**	**18**	**20**	**22**
25.1	-49	-44	-39	-33	-27	-21	-16	-10	-5	-1	**3**	**7**	**10**	**12**	**14**
27.9	-56	-51	-46	-40	-34	-28	-22	-17	-12	-8	-4	0	**3**	**5**	**8**
30.0	-61	-57	-51	-45	-40	-34	-28	-23	-18	-13	-9	-6	-3	0	**2**

Table 1: NPV of Expected Change in Profits/hectare: Base Case

[10] The results will be somewhat biased downward because only 15 years of expected gains are included in the capitalized value.

The standard NPV investment rule for a risk-neutral farmer implies that the carbon contract should be signed as long as the NPV is positive. The rule implicitly assumes that the farmer has a now-or-never opportunity to sign the contract at date 0. The focus of this section has been on the case where the risk-neutral farmer has the option to defer signing the contract. In this case, theory predicts that the NPV must be sufficiently positive (i.e. exceed the value of the option to delay) before signing the contract at date 0 is recommended. The method for calculating the value of the implicit option has been discussed.

The simulation results can also be presented graphically. Figure 5 shows the optimal investment rule for the base case parameters. The top shaded region shows the combination of parameter values for which immediate signing is recommended. The bottom shaded region shows the combination of parameter values for which immediate signing is not recommended because the NPV is negative. The middle non-shaded region is of most interest because it shows the area where signing the contract at date 0 is not recommended despite a positive NPV. It is in this region that accounting for the option value is of crucial importance.

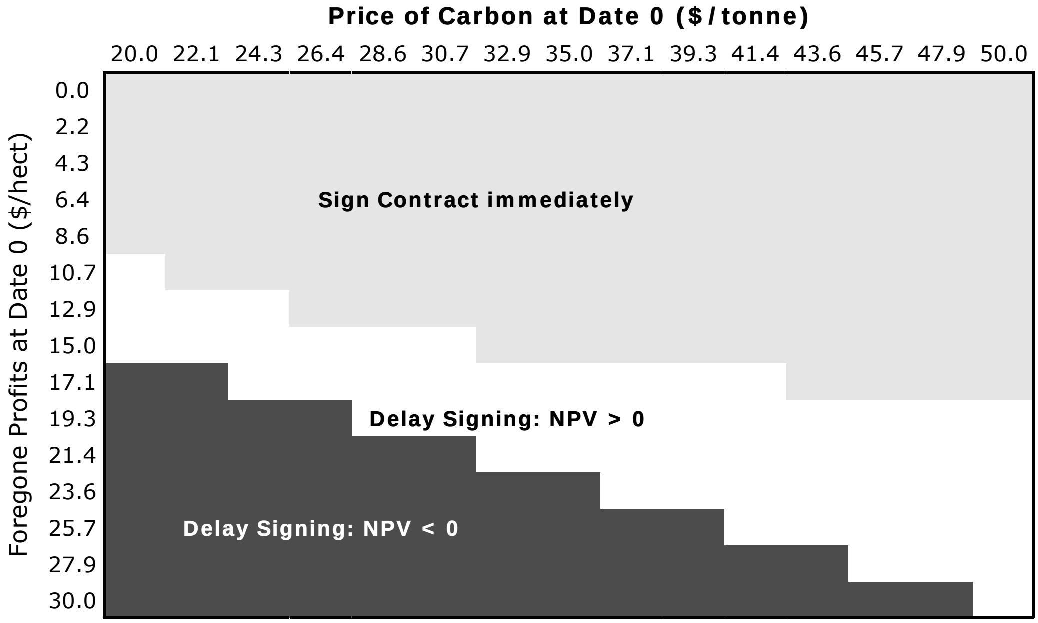

Figure 5: Optimal Contracting Decision for Base Case Parameters

For example, if the initial price of carbon is $35/ tonne and the initial level of foregone profits is $17/hectare, then the contract should not be signed despite a $31/hectare NPV. This NPV can be interpreted as the implied value of the option to delay the decision. In this particular case, the option value represents about 17 percent of the NPV of the expected stream of foregone profits associated with signing the contract.[11] The shaded area in Table 1 shows the value of the option for different parameter combinations. Note the value of the option tends to remain in the $0/hectare to $31/hectare range.

4.3 Sensitivity Analysis

The purpose of the sensitivity analysis is to examine the sensitivity of the option value results to changes in various parameters of the model. Decreasing the variability of carbon accumulation has only a small impact on the NPV of signing the carbon contract. Table 2 shows the results when the variability parameter in the soil carbon equation σ_C is decreased from 0.04 to 0.01. The main impact from this reduction in variability is to reduce the size of the shaded region (i.e. to diminish the value of the option). This result is expected because the variability in the flow of profits, combined with the irreversibility of the investment, is what yields the option value. Thus, less variability will generally result in a smaller option value. This general result is also illustrated by Table 3, in which case, beginning with base case parameters, the parameter that controls the variability of foregone profits σ_π has been reduced from 0.300 to 0.005.

11. The NPV of yearly foregone profits equal to $17/hectare over 15 years is about $185/hectare. The 17 percent ratio is calculated as 31/185.

Foregone Profits at Date 0 ($/hect)	Price of Carbon at Date 0 ($/tonne)														
	20.0	22.1	24.3	26.4	28.6	30.7	32.9	35.0	37.1	39.3	41.4	43.6	45.7	47.9	50.0
0.0	147	150	155	159	164	169	173	178	182	185	188	191	193	195	197
2.2	125	128	133	137	142	147	151	156	160	163	166	169	171	173	175
4.3	103	107	111	116	121	125	130	134	138	141	145	147	150	152	154
6.4	82	86	90	95	100	104	109	113	117	120	124	126	129	131	133
8.6	62	66	70	75	80	84	89	93	97	100	104	106	109	111	112
10.7	43	47	51	56	61	65	70	74	78	82	85	87	90	92	94
12.9	26	29	34	38.5	43	48	52	57	61	64	67	70	72	74	76
15.0	10	13	18	22	27	32	36	41	45	48	51	54	56	58	60
17.1	-5	-1	3	8	13	17	22	26	30	34	37	40	42	44	46
19.3	-18	-14	-10	-5	0	5	9	13	17	21	24	27	29	31	33
21.4	-29	-25	-21	-16	-11	-7	-2	2	6	10	13	15	18	20	22
23.6	-38	-35	-30	-26	-21	-16	-12	-7	-4	0	3	6	8	10	12
25.7	-47	-43	-38	-34	-29	-24	-20	-15	-12	-8	-5	-2	0	2	4
27.9	-53	-49	-45	-40	-36	-31	-26	-22	-18	-15	-12	-9	-7	-4	-3
30.0	-59	-55	-51	-46	-41	-37	-32	-28	-24	-20	-17	-15	-12	-10	-8

Table 2: NPV of Expected Change in Profits/hectare: Low Variability in C(t)

Foregone Profits at Date 0 ($/hect)	Price of Carbon at Date 0 ($/tonne)														
	20.0	22.1	24.3	26.4	28.6	30.7	32.9	35.0	37.1	39.3	41.4	43.6	45.7	47.9	50.0
0.0	144	149	154	160	166	172	177	183	188	192	196	200	203	205	207
2.2	122	127	132	138	144	150	156	161	166	170	174	178	181	183	186
4.3	100	105	110	116	122	128	134	139	144	148	152	156	159	161	164
6.4	78	83	88	94	100	106	112	117	122	126	130	134	137	139	142
8.6	56	61	66	72	78	84	90	95	100	104	108	112	115	117	120
10.7	35	39	45	50	56	62	68	73	78	82	86	90	93	96	98
12.9	13	17	23	28	34	40	46	51	56	60	64	68	71	74	76
15.0	-9	-5	1	6	12	18	24	29	34	39	42	46	49	52	54
17.1	-31	-27	-21	-16	-10	-4	2	7	12	17	21	24	27	30	32
19.3	-53	-49	-43	-38	-32	-26	-20	-15	-10	-5	-1	2	5	8	10
21.4	-75	-71	-65	-59	-54	-48	-42	-37	-32	-27	-23	-20	-17	-14	-12
23.6	-97	-93	-87	-81	-75	-70	-64	-59	-54	-49	-45	-42	-39	-36	-34
25.7	-119	-114	-109	-103	-97	-92	-86	-81	-76	-71	-67	-64	-61	-58	-56
27.9	-141	-136	-131	-125	-119	-114	-108	-103	-98	-93	-89	-86	-83	-80	-78
30.0	-163	-158	-153	-147	-141	-135	-130	-124	-120	-115	-111	-108	-105	-102	-100

Table 3: NPV of Expected Change in Profits/hectare: Low Variability in π(t)

Table 4 shows the results when *M* (i.e. the carbon carrying capacity of the soil when the new technology is adopted) is increased from 60 to 80 and μ_C is increased from 0.10 to 0.15. (All other parameters are set equal to base case values.) As expected, this increase in *M* increases the NPV of the carbon contract because the potential for accumulation is comparatively higher. Note also that the shaded area has increased in height. This

Foregone Profits at Date 0 ($/hect)	Price of Carbon at Date 0 ($/tonne)														
	20.0	22.1	24.3	26.4	28.6	30.7	32.9	35.0	37.1	39.3	41.4	43.6	45.7	47.9	50.0
0.0	495	508	523	539	556	572	588	603	616	628	639	649	657	664	670
2.2	437	450	465	481	497	514	529	544	558	570	581	590	598	605	612
4.3	379	392	407	423	440	456	472	486	500	512	523	532	541	548	554
6.4	323	336	351	367	384	400	416	430	444	456	467	476	485	492	498
8.6	269	283	298	314	330	347	362	377	390	403	413	423	431	438	444
10.7	219	232	247	263	280	296	312	327	340	352	363	373	381	388	394
12.9	173	186	201	217	233	250	265	280	294	306	317	326	334	341	347
15.0	130	143	158	174	191	207	223	237	251	263	274	283	292	299	305
17.1	91	104	120	136	152	168	184	199	212	225	235	245	253	260	266
19.3	57	70	85	101	118	134	150	164	178	190	201	210	219	226	232
21.4	27	40	55	71	88	104	120	135	148	160	171	181	189	196	202
23.6	1	15	30	46	62	79	94	109	123	135	146	155	163	170	176
25.7	-20	-7	8	24	41	57	73	88	101	113	124	134	142	149	155
27.9	-38	-25	-10	6	23	39	55	70	83	95	106	116	124	131	137
30.0	-53	-40	-25	9	8	24	40	54	68	80	91	100	109	116	122

Table 4: NPV of Expected Change in Profits/hectare: Large Carrying Capacity.

result implies that the range of value for which the contract should not be signed has expanded, even though the NPV is positive. This increase in the option value results because with a higher value for *M,* the absolute variation in revenues from the carbon contract is relatively higher, especially for more distant time periods.

Table 5 reports the results for the case where the discount rate has been decreased from 0.95 to 0.80. The higher discount rate of course lowers the NPV but it also decreases the size of the option value. This result is expected because a higher discount rates implies relatively less weight on distant variability in the various state variables and the variability in the state variables is highest the greatest the distance into the future. Finally, Table 6 shows the results when the contract price of carbon is assumed to grow over time. Specifically, μ_P is increased from zero to 0.015. The growth in the price of carbon of course increases the NPV of the carbon contract. More interesting is the fact that size of the option value increases as the date 0 price of carbon increases. Presumably the reason for this result is that with a higher initial carbon price, the effects of the variability in the carbon price are magnified.

Foregone Profits at Date 0 ($/hect)	Price of Carbon at Date 0 ($/tonne)														
	20.0	22.1	24.3	26.4	28.6	30.7	32.9	35.0	37.1	39.3	41.4	43.6	45.7	47.9	50.0
0.0	75	78	81	85	89	93	97	101	104	107	110	112	115	116	118
2.2	64	68	71	75	79	83	87	91	94	97	100	102	104	106	108
4.3	54	57	61	65	69	73	77	80	84	87	90	92	94	96	98
6.4	44	47	51	55	59	63	67	70	74	77	80	82	84	86	87
8.6	34	37	41	45	49	53	57	60	64	67	70	72	74	76	78
10.7	25	28	31	35	39	43	47	51	54	57	60	62	65	66	68
12.9	15	19	22	26	30	34	38	42	45	48	51	53	55	57	59
15.0	7	10	13	17	21	25	29	33	36	39	42	45	47	48	50
17.1	-1	2	5	9	13	17	21	25	28	31	34	36	38	40	42
19.3	-9	-6	-2	2	6	10	13	17	21	24	26	29	31	33	34
21.4	-16	-13	-9	-5	-1	3	7	10	14	17	19	22	24	26	27
23.6	-22	-19	-15	-11	-7	-3	1	4	8	11	13	16	18	20	21
25.7	-27	-24	-21	-17	-13	-9	-5	-1	2	5	8	11	13	14	16
27.9	-32	-29	-25	-21	-17	-13	-9	-6	-2	1	3	6	8	10	11
30.0	-36	-33	-29	-25	-21	-17	-13	-10	-6	-3	-1	2	4	6	7

Table 5: NPV of Expected Change in Profits/hectare: High Discount Rate

Foregone Profits at Date 0 ($/hect)	Price of Carbon at Date 0 ($/tonne)														
	20.0	22.1	24.3	26.4	28.6	30.7	32.9	35.0	37.1	39.3	41.4	43.6	45.7	47.9	50.0
0.0	113	126	139	153	166	179	191	204	217	229	241	253	263	270	274
2.2	91	104	118	131	144	157	169	182	195	207	219	231	241	249	252
4.3	70	83	96	109	122	135	148	161	173	186	198	209	219	227	231
6.4	49	62	75	88	101	114	127	140	152	165	177	188	198	206	210
8.6	29	42	55	68	81	94	107	120	132	145	157	168	178	186	190
10.7	10	23	36	49	62	75	88	101	113	126	138	149	159	167	171
12.9	-8	5	19	32	45	58	70	83	96	108	120	132	142	150	153
15.0	-24	-11	3	16	29	42	54	67	80	92	104	116	126	134	137
17.1	-38	-25	-12	1	14	27	40	53	65	78	90	101	111	119	123
19.3	-51	-38	-25	-12	1	14	27	40	53	65	77	88	98	106	110
21.4	-62	-49	-36	-23	-10	3	16	29	41	54	66	77	87	95	99
23.6	-72	-59	-46	-32	-19	-7	6	19	32	44	56	68	78	85	89
25.7	-80	-67	-54	-41	-28	-15	-2	11	24	36	48	60	70	77	81
27.9	-87	-74	-60	-47	-34	-21	-8	4	17	29	41	53	63	71	74
30.0	-92	-79	-66	-53	-40	-27	-14	1	11	24	36	47	57	65	69

Table 6: NPV of Expected Change in Profits/hectare: Growth in Carbon's Price

Section 5: Risk Aversion and Prudence

The previous model is missing an important component of decision making by farmers: risk aversion and prudence. It is beyond the scope of this study to examine formally the implications of risk aversion and prudence for the carbon sequestration decision. However, it is reasonable to conclude that if the decision to sign a carbon contract and to adopt a carbon accumulation management strategy results in more risk for the farm, then the higher the degree of risk aversion and prudence for the farm manager, the less likely the manager will sign the contract. In general, a risk-averse farmer is willing to sacrifice some expected profits in exchange for reduced risk. To assess fully the carbon sequestration decision, it is therefore important to determine the extent that a shift toward increased carbon accumulation affects both the change in expected profits and the change in the variability of profits. The remainder of Section 5 is devoted to a discussion about how the risk facing a farmer is likely to change as a result of the decision to sequester carbon.

The technology and management regimes that are currently used by farmers depend on a complex set of variables including expected profits, aversion to risk, previous sunk costs and "habit." The focus here is the relationship between the choice of technology and management regime and farm-level risk. Consider the farmer's choice of fertilizer application. As a result of uncertain yields and prices, theory predicts that farmers will generally apply less fertilizer than the level that would be optimal in the absence of risk. Higher rates of fertilizer will therefore increase both the expected return and the variability of return. The decision to sequester carbon by applying higher rates of fertilizer will depend on the extent that the variability of returns increases as a result of the higher rate of fertilization.

Similarly, in many farming regions, a farmer's decision to incorporate summerfallow in the crop rotation is largely a risk-management strategy. In some cases, summerfallow is likely to have important risk-reducing features and in other cases, the risk benefits of summerfallow are likely to be low to moderate. The decision to eliminate summerfallow in an attempt to sequester carbon will normally be highly dependent on the magnitude of the risk benefits attached to the summerfallow strategy.

The decision to incorporate a legume into the crop rotation also might increase risk. For example, undersowing wheat with clover so that the clover can be cut for feed following the wheat harvest might raise expected profits and result in significant increases in soil carbon.

However, during the year following the clover crop, the soil moisture in the land tends to be below average and thus subsequent crops are more susceptible to drought. Incorporating non-conventional crops into a rotation can reduce risk because of enhanced diversification, but often risk increases because markets for non-conventional crops tend to be more volatile.

There are other sources of risk besides production risk. Switching technologies often requires an investment in additional farm equipment and a portion of this investment expense is normally sunk. As with any investment that involves an up-front cost and a stream of benefits that accrue gradually over time, the farm's financial risk will increase if the investment is financed with debt. Farmers, with farms that are moderately to highly leveraged, often hesitate to take on additional debt unless the rate of return of the investment is very attractive. The decision to sequester carbon will certainly increase the degree of financial risk for many farmers and thus the extent that these farmers will be willing to sign a carbon contract will depend on their attitudes toward financial risk.

Farm financial risk is closely related to the notion of prudence. If one views the farm in the context of a lifetime consumption savings model, then the notion of prudence is central. A prudent farmer saves more than a non-prudent farmer as a contingency for future uncertainty. If capital and labour markets work efficiently, then the household consumption decisions are separate from the farm-level investment and production decisions. In reality, there are many market imperfections and thus consumption, savings and investment decisions are often highly interdependent. This interdependency suggests that the decision to invest in a carbon-accumulating technology will depend on the consumption decisions of the farm family and their desire to save. If the benefits from carbon sequestration accrue only gradually over time and the investment cost is relatively large, then a prudent farmer will be enticed to adopt the carbon strategy only if its profitability is relatively high.

There are two additional sources of risk that will emerge when a farmer chooses to sequester carbon:

- The rate of gain in stocks of soil carbon is generally highly variable and consequently the volume of carbon that is sequestered and which forms the basis of payment in the carbon contract is also highly uncertain.

- The market value of the sequestered carbon is likely to fluctuate considerably, especially while markets are being established.

Together, these two effects imply that the stream of revenues from the carbon contract are likely to be highly uncertain and thus discounted rather heavily by a risk-averse and prudent farmer. Finally, the irreversibility of the carbon sequestration decision will likely magnify the farmer's degree of risk aversion and prudence.

Section 6: Summary and Limitations

Because of extensive dynamic linkages, many sources of uncertainty and decisions that are costly to reverse, the soil carbon sequestration decision facing a farmer is highly complex. The approach taken in this report was to divide this decision into three simplifying components for the purpose of formal analysis:

- What type of technology would the farmer choose to adopt in a world with certainty?
- When would a particular technology be adopted in a world with uncertainty?
- To what extent does the adoption decision change the variability in farm profits and how does this change affect the adoption decision of a risk-averse/prudent farmer?

The model developed to answer the first question draws heavily on the standard principles of renewable resource management. The model developed to answer the second question draws heavily on the standard principles of option pricing from the finance literature on investment under uncertainty. To analyze the third question, a general discussion rather than a specific model was used.

In the first model, where uncertainty was suppressed and the focus was on the choice of technology, a specific technology gives rise to a particular long-run sustainable crop yield and a particular level of annual sustainable profits. At the one extreme, carbon-preserving technologies exist that maximize annual sustainable profits. At the other extreme, technologies exist that maximize the short-run gain from carbon depletion. The farmer's discount rate is a key determinant of where the farmer's optimal choice of technology lies between these two extremes. A carbon contract will induce the farmer to the choice of shift technology away from one that is relatively carbon depleting toward (and possibly beyond) one that maximizes annual sustainable profit. The degree of this shift depends on the selling price of carbon, the length of time that revenues will accrue from the contract, and the discount rate. Because of the simple form of the model, it is always optimal to accumulate carbon as fast as possible until the long-term equilibrium level of carbon is reached.

With the second model, where the focus was on the timing of the adoption decision, the rate of carbon accumulation, the market price of carbon, and the opportunity cost of working with the alternative technology are all random variables. Uncertainty, combined with the irreversibility of the decision, implies that the option to defer the decision to sign the contract generally has some value. The contract should be signed only if the expected NPV of the new stream of profits, less any up-front investment costs, exceeds the value of the option. The problem is solved using stochastic dynamic programming techniques.

Simulation results from the second model identify three zones:

- Zone 1-combinations of carbon contract prices and levels of foregone profits which are consistent with "sign immediately"
- Zone 2-defer signing even though NPV is positive
- Zone 3-defer signing because NPV is negative.

Zone 2 is of considerable interest. In base case simulations, Zone 2 is quite large. Option values range from $0/hectare to about $30/hectare. A $30 option value is about 15 percent of the NPV of the average cost of technology adoption. The size of the option value is lower with less uncertainty in the rate of carbon accumulation or the opportunity cost of the technology adoption. This option value is also lower, the higher the rate of discount. On the other hand, the size of the option value is higher, the greater the potential for soil carbon accumulation before the soil's carrying capacity is reached.

In general, farmers choose management strategies both to increase expected return and to decrease risk. The greater the level of risk aversion for a particular farmer, the greater the emphasis on the risk-reducing benefit of a technology. If a farmer is to be induced to sign a carbon contract and thereby to abandon a particular technology such as summerfallow or conventional tillage, then to understand fully the adoption decision, it is important to assess the extent that the switch in technology will increase the variability in farm returns. It seems that in most situations, adoption of a carbon sequestration technology will tend to increase rather than to decrease the variability of farm returns. In addition to risk aversion, prudent farmers save more when uncertainty increases. In many cases, it will be necessary to understand the farm household's entire consumption, saving and investment decision process before fully understanding the carbon sequestration adoption process.

The analysis rests upon many strong assumptions. Implications of these assumptions should be thoroughly explored in future analysis. There are twelve areas of particular concern:

- Within the model, the stochastic link between technology choice and rate of soil carbon accumulation is assumed to be well understood.
- The modeling procedure assumes that soil carbon accumulation can be accurately measured and that any natural depletion in carbon would result in a negative contract payment to the farmer.
- The assumption that the decision to sequester carbon is completely irreversible is quite unrealistic.
- The first model focuses on the long-run (sustainable) equilibrium outcome. The analysis of the path from the current level of soil carbon to the new long-run equilibrium level is quite weak.
- A household model that is based on utility maximization and which explicitly accounts for capital/debt accumulation and intergeneration transfers would be more appropriate.
- Land heterogeneity should be explicitly incorporated into the model. The incentive to sequester carbon will diminish as the base of marginal land under contract increases.
- Agro-forestry considerations should be more explicitly analyzed and discussed.

- A thorough analysis of the role of risk aversion and prudence is needed. The risk-neutrality assumption that underlies the option value model is very strong.

- In the option value model, the appropriateness of the equations that govern how soil carbon, the price of sequestered carbon and foregone profits stochastically evolve over time should be analyzed.

- The solution procedure for the option pricing model (e.g. assuming 15 discrete intervals for each state variable and a 15-year time horizon) is not very precise.

- More thorough sensitivity analysis and better intuition for the sensitivity results of the option value model is needed.

- A model with uncertainty that combines both the decision of what technology to choose and when to choose it is probably most appropriate.

In addition to these areas of particular concern, a much more thorough review of the appropriate literature is needed. The literature on the technical relationships between soil carbon and technology choice and on technology adoption is of particular importance. As well, detailed empirical work is needed to obtain realistic parameter estimates for specific scenarios. Results from the current unique generic situation are illustrative but not exceptionally informative. In addition to obtaining empirical estimates from scientific studies, farmers should be surveyed and asked to identify factors that would affect their decision to sign a carbon contract and factors that would affect their choice of adoption. Survey data should help guide the development of future models.

In conclusion, the general structure of the model should be critically analyzed. For example, a model based on the Monte Carlo simulation may be more appropriate because it would allow key results (e.g. the NPV of adoption) to be expressed as a distribution rather than a point estimate. Other types of models such as chance constrained optimization, multi-objective dynamic programming and fuzzy logic could also be analyzed for their appropriateness. The possibility of modifying existing farm management models rather than building a new model should be thoroughly examined. Any model that is developed should be robust, transparent and well documented such that a variety of research groups can use it. Finally, results of the model should be presented for many different representative situations to allow for national aggregation. National aggregation would allow policy makers to assess the overall potential of a carbon sequestration scheme for agriculture.

References

Agriculture and Agri-Food Canada. "The Health of Our Air: Toward Sustainable Agriculture in Canada." Compiled and edited by H.H. Janzen, R.L. Desjardins, J.M.R. Asselin, and B. Grace. Ottawa: Research Branch (electronic version, May 1999).

Clark, C.W. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources.* New York: John Wiley & Sons, 1976.

Dixit, A.K., and R.S. Pindyck. *Investment under Uncertainty.* Princeton, NJ: Princeton University Press, 1994.

Feng, H., J. Zhao, and C. Kling. "Carbon Sequestration in Agriculture: Value and Implementation." Working Paper 00-WP 256. Center for Agricultural and Rural Development, Iowa State University, September 2000.

Sandor, R., and J.R. Skees. "Creating a Market for Carbon Emissions." *Choices.* First Quarter 1999: 13-17.

Woodward, R.T. "Market-based Solutions to Environmental Problems: Discussion." *Journal of Agricultural and Applied Economics* 32 (August 2000): 259-266.

Appendix: Visual Basic (MS Excel) Code for Stochastic Dynamic Programming Model

```
Dim size_prof, size_valu, size_carbon, prof_L, prof_H As Double
Dim valu_L, valu_H, carbon_L, carbon_H, drift_prof, drift_valu As Double
Dim drift_carbon, sd_prof, sd_valu, sd_carbon, K_carbon As Double
Dim prof, valu, carbon, prof, valu_, carbon_ As Double
Dim T, r, delta, step_prof, step_valu, step_carbon, from_prof, from_valu, from_carbon As Double
Dim dum_print, dum_prob_add As Double
Dim prob_prof() As Single
Dim prob_valu() As Single
Dim prob_carbon() As Single
'Dim pi() As Single
Dim VO As Single
Dim F_() As Single
Dim E_prof() As Single
Dim E_valu() As Single
Dim E_carbon() As Single
Dim prob_() As Single
Sub Main_()
    'calculate the probability transition matrix
    Read_Param
    Resize_
    step_prof = (prof_H - prof_L) / (size_prof - 1)
    step_valu = (valu_H - valu_L) / (size_valu - 1)
    step_carbon = (carbon_H - carbon_L) / (size_carbon - 1)
    For i = 1 To size_prof
        prof = prof_L + (i - 1) * step_prof
    For j =1 To size_valu
        valu = valu_L + (j - 1) * step_valu
    For k =1 To size_carbon
        carbon = carbon_L + (k - 1) * step_carbon
        For i_ = 2 To size_prof -1
                prof_ = prof_L + (i_ - 1) * step_prof + 0.5 * step_prof
                prob_prof(i_) = FI_(prof, (1 + drift_prof) * prof, sd_prof * prof) - FI_(prof_ - step_prof, (1
                + drift_prof) * prof, sd_prof * prof)
```

```
    Next i_
        prob_prof(1) = FI_(prof_L + 0.5 * step_prof, (1 + drift_prof) * prof, sd_prof * prof)
        prob_prof(size_prof) = 1 - FI_(prof_H - 0.5 * step_prof, (1 + drift_prof) * prof, sd_prof *
        prof)
    For j_ = 2 To size_valu -1
        valu_ = valu_L + (j_ - 1) * step_valu + 0.5 * step_valu
        prob_valu(j_) = FI_(valu_, (1 + drift_valu) * valu, sd_valu * valu) - FI_(valu_ - step_valu,
        (1 + drift_valu) * valu, sd_valu * valu)
    Next j_
        prob_valu(1) = FI_(valu_L + 0.5 * step_valu, (1 + drift_valu) * valu, sd_valu * valu)
        prob_valu(size_valu) = 1 - FI_(valu_H - 0.5 * step_valu, (1 + drift_valu) * valu, sd_valu *
        valu)
    For k_ = 2 To size_carbon -1
        carbon_ = carbon_L + (k_ - 1) * step_carbon + 0.5 * step_carbon
        prob_carbon(k_) = FI_(carbon_, (1 + drift_carbon * (1 - carbon / K_carbon)) * carbon,
        sd_carbon * carbon) - FI_(carbon_ - step_carbon, (1 + drift_carbon * (1 - carbon /
        K_carbon)) * carbon, sd_carbon * carbon)
    Next k_
        prob_carbon(1) = FI_(carbon_L + 0.5 * step_carbon, (1 + drift_carbon * (1 - carbon /
        K_carbon)) * carbon, sd_carbon * carbon)
        prob_carbon(size_carbon) = 1 - FI_(carbon_H - 0.5 * step_carbon, (1 + drift_carbon * (1 -
        carbon / K_carbon)) * carbon, sd_carbon * carbon)
    For a =1 To size_prof
    For b =1 To size_valu
    For c = 1 To size_carbon
        prob_(i, j, k, a, b, c) = prob_prof(a) * prob_valu(b) * prob_carbon(c)
    Next c
    Next b
    Next a
Next k
Next j
Next i

' calculate various matrices for period T
For i = 1 To size_prof
For j = 1 To size_valu
For k =1 To size_carbon
    V(i, j, k, T) = 0
    F_(i, j, k, T) = 0
    E_prof(i, j, k, T) _ -(prof_L + (i - 1) * step_prof)
    E_valu(i, j, k, T) = valu_L + (j - 1) * step_valu
    E_carbon(i, j, k, T) = carbon_L + (k - 1) * step_carbon
Next k
Next j
Next i

' calculate various matrices for t < T
For tme = 1 To T - 1
For a = 1 To size_prof
For b = 1 To size_valu
For c = 1 To size_carbon
   dum_prof = 0
   dum_valu = 0
   dum_carbon = 0
```

```
    dum_V = 0
    dum_F = 0
    dum_s = dum_s + 1
    For i = 1 To size_prof
    For j =1 To size_valu
    For k = 1 To size_carbon
        dum_prof = dum_prof + prob_(a, b, c, i, j, k) * E_prof(i, j, k, T + 1 - tme)
        dum_valu = dum_valu + prob_(a, b, c, i, j, k) * E_valu(i, j, k, T + 1 - tme)
        dum_carbon = dum_carbon + prob_(a, b, c, i, j, k) * E_carbon(i, j, k, T + 1 - tme)
        dum_V = dum_V + prob_(a, b, c, i, j, k) * (V(i, j, k, T + 1 - tme) + ((carbon_L + (k - 1)
        step_carbon) - (carbon_L + (c - 1) * step_carbon)) * (valu_L + (j - 1) * step_valu))
        dum_F = dum_F + prob_(a, b, c, i, j, k) * F_(i, j, k, T + 1 - tme)
    Next k
    Next j
    Next i
    E_prof(a, b, c, T - tme) = dum_prof
    E_valu(a, b, c, T - tme) = dum_valu
    E_carbon(a, b, c, T - tme) = dum_carbon
    V(a, b, c, T - tme) = -(prof_L + (a - 1) * step_prof) + delta * dum_V
    If V(a, b, c, T - tme) > delta * dum_F Then
        F_(a, b, c, T - tme) = V(a, b, c, T - tme)
    Else
        F_(a, b, c, T - tme) = delta * dum_F
    End If
Next c
Next b
Next a
Next tme

' print results
If 1 < 2 Then
    Sheets("Results").Select
        For 1 =1 To size_prof
           Cells(2 + 1, 1).Value = prof_L + (1 - 1) * step_prof
           Cells(2 + size_prof + 2 + 1, 1).Value = prof_L + (1 - 1) * step_prof
           Cells(4 + 2 * size_prof + 2 + 1, 1).Value = prof_L + (1 - 1) * step_prof
        Next 1
        For m = 1 To size_valu
           Cells(2, 1 + m).Value = valu_L + (m - 1) * step_valu
           Cells(2 + size_prof + 2,1 + m).Value = valu_L + (m - 1) * step_valu
           Cells(4 + 2 * size_prof + 2,1 + m).Value = valu_L + (m - 1) * step_valu
        Next m
        For 1 =1 To size_prof
        For m = 1 To size_valu
           If V(1,m,1,1)>OThen
                If F_(1,m,1,1)-V(1,m,1,1)=OThen
                        Cells(2 + 1,1 + m).Value =1
                Else
                Cells(2 + 1, 1 + m).Value =0
                End If
           Else
                Cells(2 + 1, 1 + m).Value = 9
           End If
                Cells(2 + size_prof + 2 + 1,1 + m).Value = V(l, m, 1, 1)
```

```
            Cells(4 + 2 * size_prof + 2 + 1,1 + m).Value = F_(l, m, 1, 1)
        Next m
        Next 1

    For n =1 To size_prof
        Cells(4 * size_prof + n, 2).Value = prof_L + (n - 1) * step_prof
        Cells(4 * size_prof + n, 3).Value = E_prof(n, 1, 1, 1)
    Next n
    For m = 1 To size_valu
        Cells(4 * size_prof + m, 5).Value = valu_L + (m - 1) * step_valu
        Cells(4 * size_prof + m, 6).Value = E_valu(1, m, 1, 1)
    Next m
    For 1 = 1 To size_carbon
        Cells(4 * size_prof + 1, 8).Value = carbon_L + (1 - 1) * step_carbon
        Cells(4 * size_prof + 1, 9).Value = E_carbon(1, 1, 1, 1)
    Next 1

End If

' print full prob transition matrix
If 3 < 2 Then
    Sheets("Prob_Matrix").Select
    dum_ = 2
    For n = 1 To size_prof
    For m =1 To size_valu
    For 1 =1 To size_carbon
    For k = 1 To size_prof
    For j = 1 To size_valu
    For i = 1 To size_carbon
        dum_ = dum_ + 1
        Cells(dum_, 2).Value = prof_L + (n -1) * step_prof
        Cells(dum_, 3).Value = valu_L + (m -1) * step_valu
        Cells(dum_, 4).Value = carbon_L + (1 - 1) * step_carbon
        Cells(dum_, 5).Value = prof_L + (k - 1) * step_prof
        Cells(dum_, 6).Value = valu_L + (j - 1) * step_valu
        Cells(dum_, 7).Value = carbon_L + (i - 1) * step_carbon
        Cells(dum_, 8).Value = prob_(n, m, 1, k, j, i)
    Next i
    Next j
    Next k
    Next 1
    Next m
    Next n
End If

' print test results
dum_print = 0
If dum_print =1 Then
    Sheets("Test").Select
    dum = 2
    For n=1To3
    For m = 1 To 3
    For 1 = 1 To 3
        dum = dum + 1
```

```
    Cells(dum, 2).Value = prof_L + (n - 1) * step_prof
    Cells(dum, 3).Value = valu_L + (m - 1) * step_valu
    Cells(dum, 4).Value = carbon_L + (1 - 1) * step_carbon
    Cells(dum, 5).Value = E_prof(n, m, 1, 1)
    Cells(dum, 6).Value = E_valu(n, m, 1, 1)
    Cells(dum, 7).Value = E_carbon(n, m, 1, 1)
    Cells(dum, 8).Value = V(n, m, 1, 1)
    Cells(dum, 9).Value = F_(n, m, 1, 1)
Next l
Next m
Next n
dum = 32
For n = 1 To 3
For m = 1 To 3
For 1 = 1 To 3
For k = 1 To 3
For j = 1 To 3
For i = 1 To 3
    dum_ = dum_ + 1
    Cells(dum_, 2).Value = prof_L + (n -1) * step_prof
    Cells(dum_, 3).Value = valu_L + (m -1) * step_valu
    Cells(dum_, 4).Value = carbon_L + (1 - 1) * step_carbon
    Cells(dum_, 5).Value = prof_L + (k -1) * step_prof
    Cells(dum_, 6).Value = valu_L + (j - 1) * step_valu
    Cells(dum_, 7).Value = carbon_L + (i - 1) * step_carbon
    Cells(dum_, 8).Value = prob_(n, m, 1, k, j, i)
Next i
Next j
Next k
Next 1
Next m
Next n

End If
End Sub
Sub Read_Param()
    Sheets("parameters").Select
    size_prof = Cells(2, 2).Value
    size_valu = Cells(3, 2).Value
    size_carbon = Cells(4, 2).Value
    prof_L = Cells(5, 2).Value
    prof_H = Cells(6, 2).Value
    valu_L = Cells(7, 2).Value
    valu_H = Cells(8, 2).Value
    carbon_L = Cells(9, 2).Value
    carbon_H = Cells(10, 2).Value
    drift_prof = Cells(11, 2).Value
    drift_valu = Cells(12, 2).Value
    drift_carbon = Cells(13, 2).Values
    d_prof = Cells(14, 2).Values
    d_valu = Cells(15, 2).Values
    d_carbon = Cells(16, 2).Value
    K_carbon = Cells(17, 2).Value
    T = Cells(18, 2).Value
```

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    r = Cells(19, 2).Value
    delta = Cells(20, 2).Value
    from_prof = Cells(23, 2).Value
    from_valu = Cells(24, 2).Value
    from_carbon = Cells(25, 2).Value
    'Sheets("transition").Select

End Sub
Sub Resize_()
    ReDim prob_prof(size_prof) As Single
    ReDim prob_valu(size_valu) As Single
    ReDim prob_carbon(size_carbon) As Single
    ReDim prob_(size_prof, size_valu, size_carbon, size_prof, size_valu, size_carbon) As Single
    'ReDim pi(size_prof, size_valu, size_carbon) As Single
    ReDim V(size_prof, size_valu, size_carbon, T) As Single
    ReDim F_(size_prof, size_valu, size_carbon, T) As Single
    ReDim E_prof(size_prof, size_valu, size_carbon, T) As Single
    ReDim E_valu(size_prof, size_valu, size_carbon, T) As Single
    ReDim E_carbon(size_prof, size_valu, size_carbon, T) As Single
End Sub
'approximation formula from page 52 of "Handbook of the normal distribution"; J. Patel and C. Read
Function FI_(limit_, mean_, stdev)
    limit_s = (limit_ - mean_) / stdev
    If limits > -8 Then
        limit_s_s = (2 / 3.1415926) ^ 0.5 * limit_s * (1 + 0.044715 * limit_s ^2)
        FI_ = (1 + Exp(-2 * limit_s_s)) ^ -1
    Else
        FI_ = 0
    End If
End Function
```